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FIGURING OUT THE FOUR SUBCASES OF THE INTERRELATIONSHIPS OF THE COEFFICIENTS BY THE FIRST PLAYER PURE STRATEGY IN THE KERNEL OF A CONTINUOUS STRICTLY CONVEX ANTAGONISTIC GAME WITH THE CORRESPONDING THREE TYPES OF THE SOLUTION

As for the marketing activity problem there have been investigated the four subcases of the interrelationships of the coefficients by the first player pure strategy in the kernel of a continuous strictly convex antagonistic game. From that the corresponding three game solution types have been determined.

Як для задачі маркетингової діяльності досліджено чотири підвипадки співвідношень між коефіцієнтами при чистій стратегії першого гравця у ядрі однієї неперервної строго опуклої антагоністичної гри. З цього визначено відповідні три типи розв'язків гри.

Key words: marketing activity, strictly convex antagonistic game, game solution type.

The issue specification in the presented paper and the investigation goal designation with the game kernel

Clearly and without arguing, that there are many conflicting events and processes that are structured, formalized and investigated with the mathematical gaming and simulation modeling. An antagonistic game gives the exclusively relevant and good fitting mathematical model for making decisions in some competitive activity economical processes, where the two players are the rivals (including the marketing activity problem). Then an actual investigation goal designation in the antagonistic game is in finding all the solutions

$$\mathscr{S} = \left\{ \mathscr{X}_{\text{opt}}, \mathscr{Y}_{\text{opt}}, V_{\text{opt}} \right\} \quad (1)$$

of the convex continuous antagonistic games [1, 2], which kernel $S(x, y)$ as the surface is defined generally on the unit square

$$D_S = X \times Y = [0; 1] \times [0; 1], \quad (2)$$

where an element $x \in X = [0; 1]$ and an element $y \in Y = [0; 1]$ are the pure strategies of the first and second players respectively. There in the formula (1) the denomination V_{opt} is assigned for the game value. The optimal strategies set of the first player has been denoted as $\mathscr{X}_{\text{opt}}$, and the optimal strategies set of the second player has been denoted as $\mathscr{Y}_{\text{opt}}$. This paper investigation goal designation is to find all the solutions (1) of the continuous strictly convex antagonistic game with the kernel

$$S(x, y) = ax^2 + bx + gxy + cy + hy^2 + k, \quad (3)$$

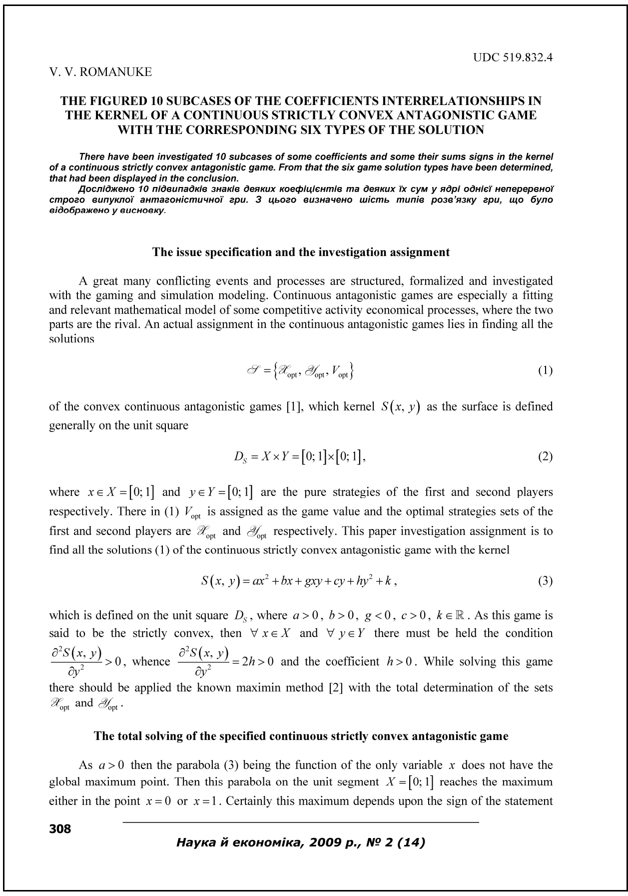
which is defined on the unit square D_S, where $a > 0$, $b < 0$, $g > 0$, $c > 0$, $k \in \mathbb{R}$. As this game has been said to be the strictly convex, then $\forall\ x \in X$ and $\forall\ y \in Y$ there must be held the condition

$$\frac{\partial^2 S(x, y)}{\partial y^2} > 0, \quad (4)$$

whence

$$\frac{\partial^2 S(x, y)}{\partial y^2} = 2h > 0 \quad (5)$$

and the coefficient $h > 0$. There should be applied the known minimax method [3, 4] with the total determining the sets $\mathscr{X}_{\text{opt}}$ and $\mathscr{Y}_{\text{opt}}$ [5 — 15], when solving this game (fig. 1 — 2).



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THE FIGURED 10 SUBCASES OF THE COEFFICIENTS INTERRELATIONSHIPS IN THE KERNEL OF A CONTINUOUS STRICTLY CONVEX ANTAGONISTIC GAME WITH THE CORRESPONDING SIX TYPES OF THE SOLUTION

There have been investigated 10 subcases of some coefficients and some their sums signs in the kernel of a continuous strictly convex antagonistic game. From that the six game solution types have been determined, that had been displayed in the conclusion.

Досліджено 10 підвипадків знаків деяких коефіцієнтів та деяких їх сум у ядрі однієї неперервної строго випуклої антагоністичної гри. З цього визначено шість типів розв'язку гри, що було відображено у висновку.

The issue specification and the investigation assignment

A great many conflicting events and processes are structured, formalized and investigated with the gaming and simulation modeling. Continuous antagonistic games are especially a fitting and relevant mathematical model of some competitive activity economical processes, where the two parts are the rival. An actual assignment in the continuous antagonistic games lies in finding all the solutions

$$\mathscr{S} = \left\{ \mathscr{X}_{\text{opt}}, \mathscr{Y}_{\text{opt}}, V_{\text{opt}} \right\} \qquad (1)$$

of the convex continuous antagonistic games [1], which kernel $S(x, y)$ as the surface is defined generally on the unit square

$$D_S = X \times Y = [0; 1] \times [0; 1], \qquad (2)$$

where $x \in X = [0; 1]$ and $y \in Y = [0; 1]$ are the pure strategies of the first and second players respectively. There in (1) V_{opt} is assigned as the game value and the optimal strategies sets of the first and second players are $\mathscr{X}_{\text{opt}}$ and $\mathscr{Y}_{\text{opt}}$ respectively. This paper investigation assignment is to find all the solutions (1) of the continuous strictly convex antagonistic game with the kernel

$$S(x, y) = ax^2 + bx + gxy + cy + hy^2 + k, \qquad (3)$$

which is defined on the unit square D_S, where $a > 0$, $b > 0$, $g < 0$, $c > 0$, $k \in \mathbb{R}$. As this game is said to be the strictly convex, then $\forall\, x \in X$ and $\forall\, y \in Y$ there must be held the condition $\frac{\partial^2 S(x, y)}{\partial y^2} > 0$, whence $\frac{\partial^2 S(x, y)}{\partial y^2} = 2h > 0$ and the coefficient $h > 0$. While solving this game there should be applied the known maximin method [2] with the total determination of the sets $\mathscr{X}_{\text{opt}}$ and $\mathscr{Y}_{\text{opt}}$.

The total solving of the specified continuous strictly convex antagonistic game

As $a > 0$ then the parabola (3) being the function of the only variable x does not have the global maximum point. Then this parabola on the unit segment $X = [0; 1]$ reaches the maximum either in the point $x = 0$ or $x = 1$. Certainly this maximum depends upon the sign of the statement

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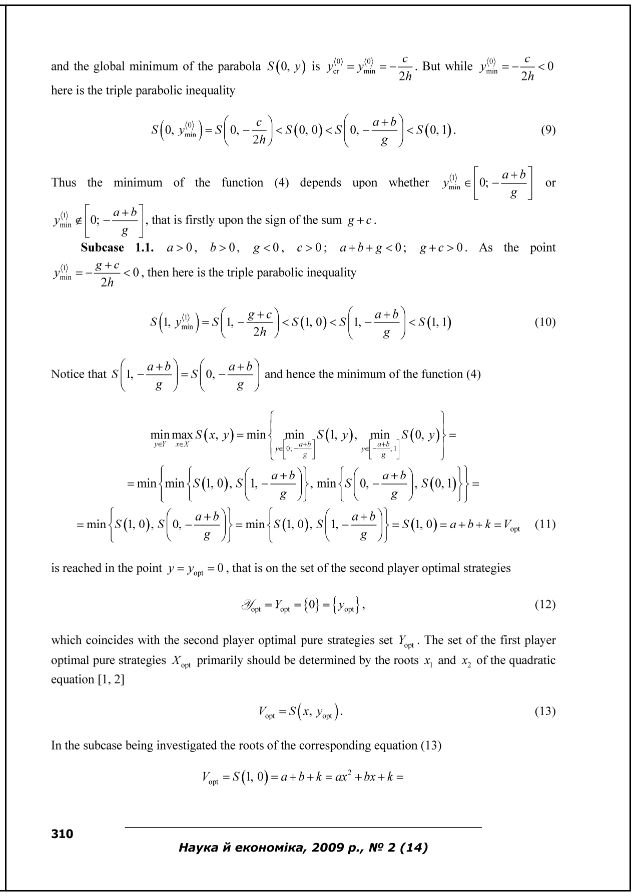
Fig. 1. The paper [1] start page



and the global minimum of the parabola $S(0, y)$ is $y_{\text{cr}}^{\langle 0 \rangle} = y_{\min}^{\langle 0 \rangle} = -\frac{c}{2h}$. But while $y_{\min}^{\langle 0 \rangle} = -\frac{c}{2h} < 0$ here is the triple parabolic inequality

$$S\left(0, y_{\min}^{\langle 0 \rangle}\right) = S\left(0, -\frac{c}{2h}\right) < S(0, 0) < S\left(0, -\frac{a+b}{g}\right) < S(0, 1). \qquad (9)$$

Thus the minimum of the function (4) depends upon whether $y_{\min}^{\langle 1 \rangle} \in \left[0; -\frac{a+b}{g}\right]$ or $y_{\min}^{\langle 1 \rangle} \notin \left[0; -\frac{a+b}{g}\right]$, that is firstly upon the sign of the sum $g + c$.

Subcase 1.1. $a > 0$, $b > 0$, $g < 0$, $c > 0$; $a + b + g < 0$; $g + c > 0$. As the point $y_{\min}^{\langle 1 \rangle} = -\frac{g+c}{2h} < 0$, then here is the triple parabolic inequality

$$S\left(1, y_{\min}^{\langle 1 \rangle}\right) = S\left(1, -\frac{g+c}{2h}\right) < S(1, 0) < S\left(1, -\frac{a+b}{g}\right) < S(1, 1) \qquad (10)$$

Notice that $S\left(1, -\frac{a+b}{g}\right) = S\left(0, -\frac{a+b}{g}\right)$ and hence the minimum of the function (4)

$$\min_{y \in Y} \max_{x \in X} S(x, y) = \min\left\{ \min_{y \in \left[0; -\frac{a+b}{g}\right]} S(1, y),\ \min_{y \in \left[-\frac{a+b}{g}; 1\right]} S(0, y) \right\} =$$
$$= \min\left\{ \min\left\{ S(1, 0), S\left(1, -\frac{a+b}{g}\right) \right\}, \min\left\{ S\left(0, -\frac{a+b}{g}\right), S(0, 1) \right\} \right\} =$$
$$= \min\left\{ S(1, 0), S\left(0, -\frac{a+b}{g}\right) \right\} = \min\left\{ S(1, 0), S\left(1, -\frac{a+b}{g}\right) \right\} = S(1, 0) = a + b + k = V_{\text{opt}} \qquad (11)$$

is reached in the point $y = y_{\text{opt}} = 0$, that is on the set of the second player optimal strategies

$$\mathscr{Y}_{\text{opt}} = Y_{\text{opt}} = \{0\} = \left\{ y_{\text{opt}} \right\}, \qquad (12)$$

which coincides with the second player optimal pure strategies set Y_{opt}. The set of the first player optimal pure strategies X_{opt} primarily should be determined by the roots x_1 and x_2 of the quadratic equation [1, 2]

$$V_{\text{opt}} = S\left(x, y_{\text{opt}}\right). \qquad (13)$$

In the subcase being investigated the roots of the corresponding equation (13)

$$V_{\text{opt}} = S(1, 0) = a + b + k = ax^2 + bx + k =$$

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Fig. 2. The minimax method in using [1]

The total thorough solving of the specified continuous strictly convex antagonistic game with the adjusted kernel

First of all, there should be marked, that as the coefficient $a > 0$ then the parabola (3), being the function of the only variable x, does not have the global maximum point. Consequently this parabola on the unit segment $X = [0; 1]$ reaches the maximum either in the point $x = 0$ or $x = 1$ and, certainly, this maximum depends upon the sign of the statement $a + b + gy$. While having $a > 0$, $b < 0$, $g > 0$, $c > 0$ then there $a + b + gy > 0$ if $y > -\frac{a+b}{g}$. The value $-\frac{a+b}{g} > 0$ when $a + b < 0$, and the value $-\frac{a+b}{g} \leqslant 1$ when $a + b + g \geqslant 0$. That will be the prime two subcases in the presented game investigation.

Subcase 1.1. $a > 0$, $b < 0$, $g > 0$, $c > 0$; $a + b < 0$; $a + b + g \geqslant 0$. As here $a + b + gy > 0$ $\forall\, y > -\frac{a+b}{g}$ by $-\frac{a+b}{g} \in (0; 1] \subset Y$ then the maximum of the surface (3) on the unit segment X of the variable x is

$$\max_{x \in X} S(x, y) = \begin{cases} S(0, y) = cy + hy^2 + k,\ y \in \left[0; -\frac{a+b}{g}\right], \\ S(1, y) = a + b + gy + cy + hy^2 + k,\ y \in \left[-\frac{a+b}{g}; 1\right]. \end{cases} \qquad (6)$$

To minimize the function (6) on the unit segment Y it is necessary that the minimum of the parabola $S(0, y)$ on the segment $\left[0; -\frac{a+b}{g}\right] \subset Y$ should be found, and also that the minimum of the parabola $S(1, y)$ on the segment $\left[-\frac{a+b}{g}; 1\right] \subset Y$ should be found. Then before finding the local minimum of the parabola $S(0, y)$ on some subsegment of the unit segment Y primarily the global minimum of the parabola $S(0, y)$ should be determined.

The first derivative of the parabola $S(0, y)$ is

$$\frac{d}{dy}S(0, y) = \frac{d}{dy}\left(cy + hy^2 + k\right) = c + 2hy \tag{7}$$

and the first critical point of the parabola $S(0, y)$ is the zero point of the line (7) $y = y_{\text{cr}}^{\langle 0\rangle} = -\frac{c}{2h}$. The second derivative of the parabola $S(0, y)$ is

$$\frac{d^2}{dy^2}S(0, y) = \frac{d}{dy}\left(c + 2hy\right) = 2h > 0 \tag{8}$$

and the global minimum of the parabola $S(0, y)$ is $y_{\text{cr}}^{\langle 0\rangle} = y_{\min}^{\langle 0\rangle} = -\frac{c}{2h}$. However, by the initial conditions, here the point $y_{\min}^{\langle 0\rangle} = -\frac{c}{2h} < 0$.

The first derivative of the parabola $S(1, y)$ is

$$\frac{d}{dy}S(1, y) = \frac{d}{dy}\left(a + b + gy + cy + hy^2 + k\right) = g + c + 2hy\,. \tag{9}$$

The first critical point of the parabola $S(1, y)$ is the zero point of the line (9), that is $y = y_{\text{cr}}^{\langle 1\rangle} = -\frac{g+c}{2h}$, and as the second derivative of the parabola $S(1, y)$ is

$$\frac{d^2}{dy^2}S(1, y) = \frac{d}{dy}\left(g + c + 2hy\right) = 2h > 0 \tag{10}$$

once again, then the global minimum of the parabola $S(1, y)$ is $y_{\text{cr}}^{\langle 1\rangle} = y_{\min}^{\langle 1\rangle} = -\frac{g+c}{2h}$. However, by the initial conditions, here the point $y_{\min}^{\langle 1\rangle} = -\frac{g+c}{2h} < 0$.

Accordingly, neither the point $y_{\min}^{\langle 0\rangle} = -\frac{c}{2h} \in Y$ nor the point $y_{\min}^{\langle 1\rangle} = -\frac{g+c}{2h} \in Y$, that is

$$\left\{y_{\min}^{\langle 0\rangle}, y_{\min}^{\langle 1\rangle}\right\} = \left\{-\frac{c}{2h}, -\frac{g+c}{2h}\right\} \not\subset Y\,. \tag{11}$$

But as the point $y_{\min}^{\langle 0\rangle} = -\frac{c}{2h} < 0$ then with the condition $-\frac{a+b}{g} \in (0; 1] \subset Y$ here is the triple parabolic inequality

$$S\left(0, y_{\min}^{\langle 0\rangle}\right) = S\left(0, -\frac{c}{2h}\right) < S(0, 0) < S\left(0, -\frac{a+b}{g}\right) \leqslant S(0, 1)\,. \tag{12}$$

And as the point $y_{\min}^{\langle 1\rangle} = -\frac{g+c}{2h} < 0$ then with the condition $-\frac{a+b}{g} \in (0; 1] \subset Y$ here is again the triple parabolic inequality

$$S\left(1,\, y_{\min}^{\langle 1\rangle}\right) = S\left(1,\, -\frac{g+c}{2h}\right) < S(1,\, 0) < S\left(1,\, -\frac{a+b}{g}\right) \leqslant S(1,\, 1)\,. \tag{13}$$

With the obvious identity

$$S\left(1,\, -\frac{a+b}{g}\right) = S\left(0,\, -\frac{a+b}{g}\right) \tag{14}$$

and the inequalities (12) and (13) the minimum of the function (6)

$$\begin{gathered} \min_{y\in Y}\max_{x\in X} S(x,\, y) = \min\left\{ \min_{y\in\left[0;\, -\frac{a+b}{g}\right]} S(0,\, y),\ \min_{y\in\left[-\frac{a+b}{g};\, 1\right]} S(1,\, y) \right\} = \\ = \min\left\{ \min\left\{ S(0,\, 0),\, S\left(0,\, -\frac{a+b}{g}\right) \right\},\, \min\left\{ S\left(1,\, -\frac{a+b}{g}\right),\, S(1,\, 1) \right\} \right\} = \\ = \min\left\{ S(0,\, 0),\, S\left(1,\, -\frac{a+b}{g}\right) \right\} = \min\left\{ S(0,\, 0),\, S\left(0,\, -\frac{a+b}{g}\right) \right\} = S(0,\, 0) = k = V_{\mathrm{opt}} \end{gathered} \tag{15}$$

is reached in the point $y = y_{\mathrm{opt}} = 0$, that is on the set of the second player optimal strategies

$$\mathscr{Y}_{\mathrm{opt}} = Y_{\mathrm{opt}} = \{0\} = \left\{ y_{\mathrm{opt}} \right\}, \tag{16}$$

which coincides with the second player optimal pure strategies set Y_{opt}. The set of the first player optimal pure strategies X_{opt} primarily should be determined by the roots x_1 and x_2 of the quadratic equation [1 — 15]

$$V_{\mathrm{opt}} = S\left(x,\, y_{\mathrm{opt}}\right). \tag{17}$$

Straightly hereon the corresponding equation (17) is

$$V_{\mathrm{opt}} = S(0,\, 0) = k = ax^2 + bx + k = ax\left(x + \frac{b}{a}\right) + k = S(x,\, 0) = S\left(x,\, y_{\mathrm{opt}}\right). \tag{18}$$

The roots of the equation (18) are $x_1 = 0$ and $x_2 = -\frac{b}{a}$. But $a+b<0$ means that $-\frac{b}{a} > 1$. Then $x_1 \in X$, $x_2 \notin X$. Thereupon the set

$$X_{\mathrm{opt}} = \{x_1\} = \{0\} = \mathscr{X}_{\mathrm{opt}} \tag{19}$$

and the investigated subcase (fig. 3) game solution is the set

$$\mathscr{S} = \left\{ \{0\},\, \{0\},\, k \right\}. \tag{20}$$

Subcase 1.2. $a>0$, $b<0$, $g>0$, $c>0$; $a+b<0$; $a+b+g<0$. As the value $-\frac{a+b}{g} > 1$ then here $a+b+gy<0 \quad \forall\, y < -\frac{a+b}{g}$ or $\forall\, y \in [0;\, 1]$ and the maximum of the surface (3) on the unit segment X of the variable x is the parabola

$$\max_{x\in X} S(x,\, y) = \max\left\{S(0,\, y),\, S(1,\, y)\right\} = S(0,\, y) = cy + hy^2 + k\,. \tag{21}$$

As the point $y_{\min}^{\langle 0\rangle} = -\dfrac{c}{2h} < 0$ then there is the double parabolic inequality

$$S\left(0,\, y_{\min}^{\langle 0\rangle}\right) = S\left(0,\, -\frac{c}{2h}\right) < S(0,\, 0) < S(0,\, 1)\,. \tag{22}$$

Hence the minimum of the parabola (21)

$$\min_{y\in Y}\max_{x\in X} S(x,\, y) = \min_{y\in Y} S(0,\, y) = \min\left\{S(0,\, 0),\, S(0,\, 1)\right\} = S(0,\, 0) = k = V_{\text{opt}} \tag{23}$$

is reached on the set (16). The roots of the corresponding equation (17), which now is identical to the equation (18), are again $x_1 = 0$ and $x_2 = -\dfrac{b}{a}$. But $a + b < 0$, $-\dfrac{b}{a} > 1$ and $x_2 > 1$. Then $x_1 \in X$, $x_2 \notin X$ and there is the set (19), and the investigated subcase (fig. 4) game solution is the set (20).

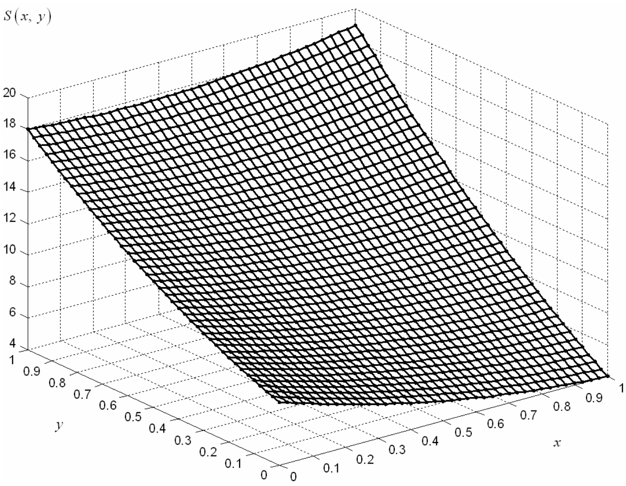

Fig. 3. The subcase $a > 0$, $b < 0$, $g > 0$, $c > 0$, $a + b < 0$, $a + b + g \geqslant 0$ **with the kernel** $S(x,\, y) = 3x^2 - 7x + 5xy + 6y + 4y^2 + 8$ **and the game solution** $\mathscr{S} = \left\{\{0\},\, \{0\},\, 8\right\}$

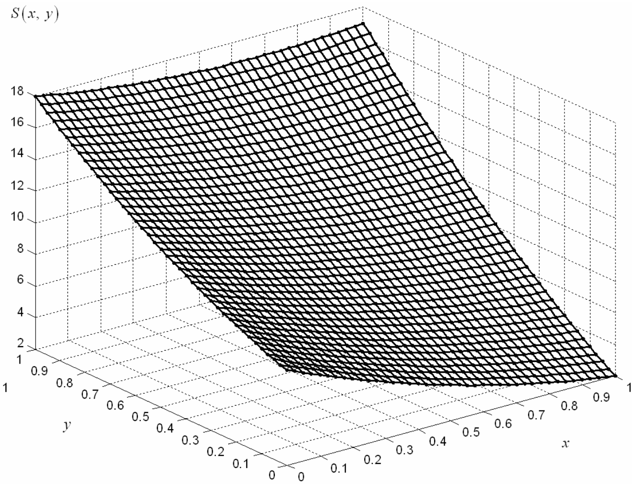

Fig. 4. The subcase $a > 0$, $b < 0$, $g > 0$, $c > 0$, $a + b < 0$, $a + b + g < 0$ **with the kernel** $S(x,\, y) = 3x^2 - 9x + 5xy + 6y + 4y^2 + 8$ **and the game solution** $\mathscr{S} = \left\{\{0\},\, \{0\},\, 8\right\}$

Subcase 2. $a > 0$, $b < 0$, $g > 0$, $c > 0$; $a + b > 0$. Obviously that here $a + b + gy > 0 \;\; \forall\, y > -\dfrac{a+b}{g}$ by $-\dfrac{a+b}{g} < 0$, whence $a + b + gy > 0 \;\; \forall\, y \in [0;\, 1]$. Then the maximum of the surface (3) on the unit segment X of the variable x is the parabola

$$\max_{x\in X} S(x,\, y) = \max\left\{S(0,\, y),\, S(1,\, y)\right\} = S(1,\, y) = a + b + gy + cy + hy^2 + k\,. \tag{24}$$

As the point $y_{\min}^{\langle 1\rangle} = -\dfrac{g+c}{2h} < 0$ then there is the double parabolic inequality

$$S\left(1,\, y_{\min}^{\langle 1\rangle}\right) = S\left(1,\, -\frac{g+c}{2h}\right) < S(1,\, 0) < S(1,\, 1)\,. \tag{25}$$

Hence the minimum of the parabola (24)

$$\min_{y\in Y}\max_{x\in X} S(x,\, y) = \min_{y\in Y} S(1,\, y) = \min\{S(1,\, 0),\, S(1,\, 1)\} = S(1,\, 0) = a+b+k = V_{\text{opt}} \tag{26}$$

is reached on the set (16). The roots of the corresponding equation (17)

$$V_{\text{opt}} = S(1,\, 0) = a+b+k = ax^2+bx+k =$$
$$= a(x-1)\left(x+\frac{a+b}{a}\right)+a+b+k = S(x,\, 0) = S(x,\, y_{\text{opt}}) \tag{27}$$

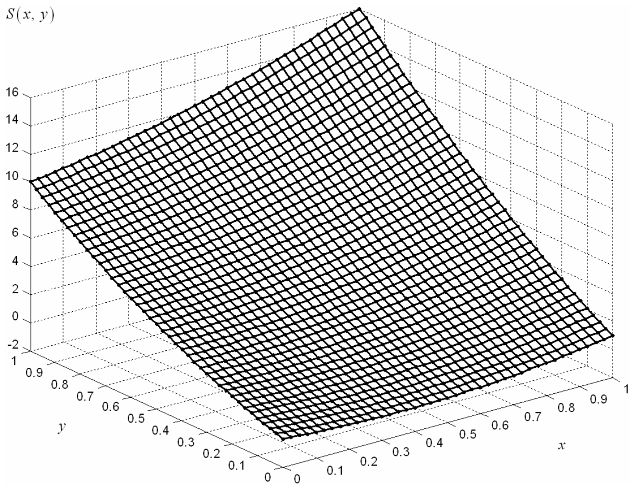

Fig. 5. The subcase $a>0$, $b<0$, $g>0$, $c>0$, $a+b>0$ **with the kernel** $S(x,\, y) = 3x^2-2x+5xy+6y+4y^2$ **and the game solution** $\mathscr{S} = \{\{1\},\, \{0\},\, 1\}$

are $x_1 = -\dfrac{a+b}{a}$ and $x_2 = 1$. However, as $-\dfrac{a+b}{a} < 0$, then $x_1 \notin X$, $x_2 \in X$ and there is the set

$$X_{\text{opt}} = \{x_2\} = \{1\} = \mathscr{X}_{\text{opt}}, \tag{28}$$

whence the investigated subcase (fig. 5) game solution is

$$\mathscr{S} = \{\{1\},\, \{0\},\, a+b+k\}. \tag{29}$$

Subcase 3. $a>0$, $b<0$, $g>0$, $c>0$; $a+b=0$. As here the inequality $a+b+gy>0$ $\forall\ y > -\dfrac{a+b}{g} = 0$ then the maximum of the surface (3) on the unit segment X of the variable x is the function

$$\max_{x\in X} S(x,\, y) = \begin{cases} S(0,\, y) = cy+hy^2+k = S(1,\, y),\ y\in\{0\}, \\ S(1,\, y) = a+b+gy+cy+hy^2+k,\ y\in(0;\,1]. \end{cases} \tag{30}$$

With the obvious identity (14), which now is

$$S\left(1,\, -\frac{a+b}{g}\right) = S(1,\, 0) = S(0,\, 0) = S\left(0,\, -\frac{a+b}{g}\right), \tag{31}$$

and the double parabolic inequality (25) the minimum of the function (30)

$$\min_{y\in Y}\max_{x\in X} S(x,\, y) = \min\left\{\min_{y\in\{0\}} S(0,\, y),\ \min_{y\in(0;\,1]} S(1,\, y)\right\} =$$
$$= \min\{S(0,\, 0),\, \min\{S(1,\, 0),\, S(1,\, 1)\}\} =$$
$$= \min\{S(0,\, 0),\, S(1,\, 0)\} = k = V_{\text{opt}} \tag{32}$$

is reached on the set (16). The roots of the corresponding equation (17), which now turns to the equation (18), are $x_1 = 0$ and $x_2 = -\dfrac{b}{a} = 1$. Then $x_1 \in X$ and $x_2 \in X$. Consequently, the set

$$X_{\text{opt}} = \{x_1,\, x_2\} = \{0,\, 1\}. \tag{33}$$

May $P(x_1)$ and $P(x_2)$ be the probabilities of the first player to select its pure strategies $x_1 = x_{\text{opt}}^{\langle 1\rangle}$ and $x_2 = x_{\text{opt}}^{\langle 2\rangle}$.

Then the set

$$\mathscr{X}_{\text{opt}} = \left\{ X_{\text{opt}}, \left\{ P\left(x_{\text{opt}}^{\langle 1 \rangle}\right), P\left(x_{\text{opt}}^{\langle 2 \rangle}\right) \right\} \right\} \tag{34}$$

and there are the statements $X_{\text{opt}} = \left\{ x_{\text{opt}}^{\langle 1 \rangle}, x_{\text{opt}}^{\langle 2 \rangle} \right\}$,

$$P\left(x_{\text{opt}}^{\langle 1 \rangle}\right) + P\left(x_{\text{opt}}^{\langle 2 \rangle}\right) = 1 . \tag{35}$$

Those probabilities satisfy the double inequality [1, 2, 6]

$$S\left(x^{\langle 1 \rangle}, y_{\text{opt}}\right) P\left(x^{\langle 1 \rangle}\right) + S\left(x^{\langle 2 \rangle}, y_{\text{opt}}\right) P\left(x^{\langle 2 \rangle}\right) \leqslant V_{\text{opt}} \leqslant S\left(x_{\text{opt}}^{\langle 1 \rangle}, y\right) P\left(x_{\text{opt}}^{\langle 1 \rangle}\right) + S\left(x_{\text{opt}}^{\langle 2 \rangle}, y\right) P\left(x_{\text{opt}}^{\langle 2 \rangle}\right), \tag{36}$$

where $y \neq y_{\text{opt}}$, and $x^{\langle 1 \rangle} \neq x_{\text{opt}}^{\langle 1 \rangle}$, or $x^{\langle 2 \rangle} \neq x_{\text{opt}}^{\langle 2 \rangle}$, or $P\left(x^{\langle 1 \rangle}\right) \neq P\left(x_{\text{opt}}^{\langle 1 \rangle}\right)$. In the being investigated subcase the probabilities $P(x_1) = P(0)$ and $P(x_2) = P(1)$ may be determined from the right inequality (36), where $y \neq y_{\text{opt}} = 0$:

$$\begin{aligned} V_{\text{opt}} = S(0, 0) = S(1, 0) = k \leqslant S(0, y) P(0) + S(1, y) P(1) = \\ = \left(cy + hy^2 + k\right) P(0) + \left(a + b + gy + cy + hy^2 + k\right) P(1) = \\ = cy + hy^2 + (a + b + gy) P(1) + k = cy + hy^2 + gyP(1) + k \,; \end{aligned} \tag{37}$$

$$0 \leqslant cy + hy^2 + gyP(1) \,; \tag{38}$$

$$gyP(1) \geqslant -cy - hy^2 . \tag{39}$$

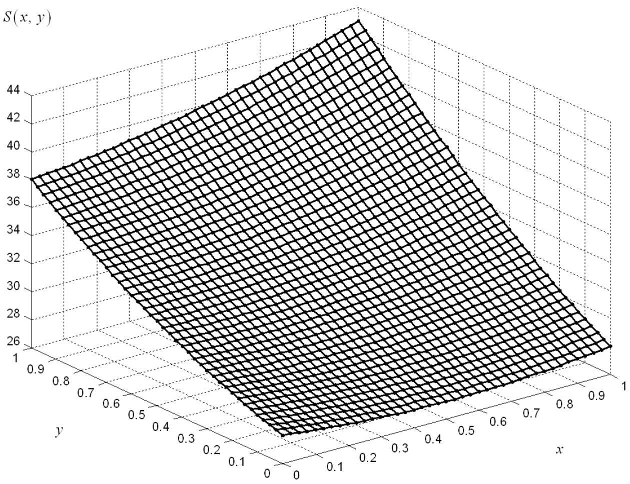

Fig. 6. The subcase $a > 0$, $b < 0$, $g > 0$, $c > 0$, $a + b = 0$ **with the kernel** $S(x, y) = 3x^2 - 3x + 5xy + 6y + 4y^2 + 28$ **and the game solution** $\mathscr{S} = \left\{ \left\{ \{0, 1\}, \left\{1 - P(1), P(1)\right\} \right\}, \{0\}, 28 \right\}$, **where** $P(1) \in [0; 1]$

As $gy > 0$ than dividing the members of the inequality (39) by gy will not change the sign in (39). Hence the probability $P(1)$ evaluation is

$$P(1) \geqslant -\frac{c + hy}{g} . \tag{40}$$

But inasmuch as $c > 0$, $h > 0$ and $g > 0$ then $-\frac{c + hy}{g} < 0 \quad \forall\, y \in (0; 1]$. Therefore the probability $P(1)$ here satisfies the condition

$$P(1) \in [0; 1] = X \tag{41}$$

and, resuming, in the investigated subcase (fig. 6) the set

$$\mathscr{X}_{\text{opt}} = \left\{ \{0, 1\}, \left\{1 - P(1), P(1)\right\} \right\} \tag{42}$$

and the game solution

$$\mathscr{S} = \left\{ \left\{ \{0, 1\}, \left\{1 - P(1), P(1)\right\} \right\}, \{0\}, k \right\} . \tag{43}$$

Conclusion

Having investigated the four subcases of the coefficients a, b and g interrelationships in the kernel (3), where the coefficients $a > 0$, $b < 0$, $g > 0$, $c > 0$, there have been determined the corresponding three types of the convex continuous antagonistic game solution. When $a+b<0$ then the game solution is the set (20). The set (29) is the game solution for the subcase with $a+b>0$. In the boundary subcase with $a+b=0$ the game solution is the set (43) with the condition (41).

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