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FAST SOLUTION OF THE DISCRETE NOISELESS DUEL WITH THE NONLINEAR SCALE ON THE LINEAR ACCURACY FUNCTIONS

There has been defined the discrete noiseless duel with the skewsymmetric kernel on the finite nonlinearly scaled subset of the unit square. The program for fast getting the defined duel solution has been embodied.

Определено дискретную бесшумную дуэль с кососимметрическим ядром на конечном нелинейно масштабированном подмножестве единичного квадрата. Воплощено программу для быстрого получения решения определённой дуэли.

Key words: discrete noiseless duel, skewsymmetric kernel, unit square, MATLAB, accuracy functions.

The problem in a general view

The simplest math model of resolving the noncooperative conflict is the antagonistic game. The time selection antagonistic games are really versatile to use them in decision making on social-economic and bio-ecological events [1]. Those duels usually are defined as infinite games, though in practice a player mostly has the finite number of its alternatives. However, the finite alternatives duel games had been investigated in less, than the class of the infinite ones. Thus the actuality of the discrete duel investigation is the highest. The noiseless duels as a class of the time selection antagonistic games are crucial because of their fundamentality, and their sampling action gives the decision making models, allowing to practice the found optimal alternatives. Besides, this sampling should regard to that the duel end is closer the importance of the strategical alternative is higher.

The near and last papers on the problem direction

Some narrowed classes of the noiseless duels had been solved or, at least, considered in [2, 3]. The corresponding discrete noiseless duels, having appeared in [4], are not highlighted. That might be caused with the reference to their easy sampling from the infinite or continuous duels, although the sampling of the continuous solution does not drive to the discrete noiseless duel solution. But naturally, that the sampled infinite duel is a matrix game, and its solution may be obtained just with some math software, working operatively in the linear programming way. And the importance of the longer time strategies must be built in there also.

The task assignments and the paper aim formulation

Firstly the kernel

$$K(x, y) = x - y + xy\,\mathrm{sign}(y - x) \tag{1}$$

of the noiseless duel on the unit square

$$X \times Y = [0; 1] \times [0; 1] \tag{2}$$

should be sampled generally in such a way, that the importance of the more lingered strategies would be greater, where $x \in X$ is a pure strategy of the first player, and $y \in Y$ is a pure strategy of the second. Not incidentally, but here the accuracy functions are taken linear, as the sampled strategy importance will be only in its specified neighborhood with the other sampled strategies. That is the density of the pure strategies on X and Y, when the time is going on, must be not decreasing. Then the obtained discrete noiseless duel, being symmetric, will be solved within the MATLAB [5]. Thus there will be obtained the fast solution of the discrete noiseless duel (1) with the nonlinear scale on the linear accuracy functions, reflecting the significance of the more lingered alternatives, and the aim of this paper will be achieved.

The noiseless duel (1) nonlinear scale sampling

The skew-symmetry of the game with the kernel (1) is obvious:

$$K(x, y) = -K(y, x).$$

This means that the optimal strategies of the players in the noiseless duel (1) are identical, and, as corollary, the game value is zero. The sampling on the unit segment $\left[0;1\right]$ scale will be the same for both players, letting them shoot in the very beginning and in the end of the duel, that is the pure strategies 0 and 1 are included into the sampled set of the pure strategies of the player.

May the probability d_j be the distance between the left adjacent (previous) pure strategy and the present, where $d_j \notin \{0, 1\}$ and $j = \overline{1, N-2}$ by the N available pure strategies at the player. Then the corresponding vector

$$\mathbf{D} = \left[d_1 \quad d_2 \quad \ldots \quad d_{N-3} \quad d_{N-2}\right] \tag{3}$$

of distances should satisfy the conditions in the following:

$$\left[d_1 \quad d_2 \quad \ldots \quad d_{N-3} \quad d_{N-2}\right] \in$$
$$\in \left\{ \mathbf{D} \in \mathbb{R}^{N-2} \mid d_j \in (0;1)\ \forall\ j = \overline{1, N-2},\ d_j \geqslant d_{j+1}\ \forall\ j = \overline{1, N-3},\ 1 - \sum_{j=1}^{N-2} d_j \leqslant d_{N-2} \right\}. \tag{4}$$

Accordingly to the distances vector (4), in the discrete noiseless duel with the nonlinear scale on the linear accuracy functions, the first player has its set

$$X_{\mathbf{D}} = \{x_k\}_{k=1}^{N} = \{0, 1\} \cup \left\{ \sum_{j=1}^{k-1} d_j \right\}_{k=2}^{N-1} \tag{5}$$

of the pure strategies, and the second player has the set

$$Y_{\mathbf{D}} = \{y_k\}_{k=1}^{N} = \{0, 1\} \cup \left\{ \sum_{j=1}^{k-1} d_j \right\}_{k=2}^{N-1} \tag{6}$$

of its pure strategies. Thus such a duel is defined on the subset

$$X_{\mathbf{D}} \times Y_{\mathbf{D}} = \{x_k\}_{k=1}^{N} \times \{y_l\}_{l=1}^{N} =$$
$$= \left\{ \left\{ [x_k \quad y_l] \right\}_{l=1}^{N} \right\}_{k=1}^{N} =$$
$$= \left\{ \{0, 1\} \cup \left\{ \sum_{j=1}^{k-1} d_j \right\}_{k=2}^{N-1} \right\} \times \left\{ \{0, 1\} \cup \left\{ \sum_{i=1}^{l-1} d_i \right\}_{l=2}^{N-1} \right\} \subset$$
$$\subset X \times Y = [0;1] \times [0;1] \tag{7}$$

of the unit square (2), satisfying the conditions in (4). And here, for the not decreasing density of the sorted pure strategies in (5) and (6), the importance of the more lingered strategies is greater, as it has been assigned.

The MATLAB fast solution code

For knowing the sampled sets (5) and (6), the sampled kernel (1) may be analytically represented as the matrix $\mathbf{F} = \left(f_{ij}\right)_{N \times N}$ with the elements

$$r_{ij} = K\left(x_i, y_j\right) =$$
$$= x_i - y_j + x_i y_j \operatorname{sign}\left(y_j - x_i\right),\ \ i = \overline{1, N},\ \ j = \overline{1, N}. \tag{8}$$

To solve this $\left(f_{ij}\right)_{N \times N}$-game there has been constructed the MATLAB code `dndns` as the independent program module (figure 1), having the distances vector (3) as the input. Some examples, demonstrating the work of the module `dndns`, are in the figures 2 — 4.

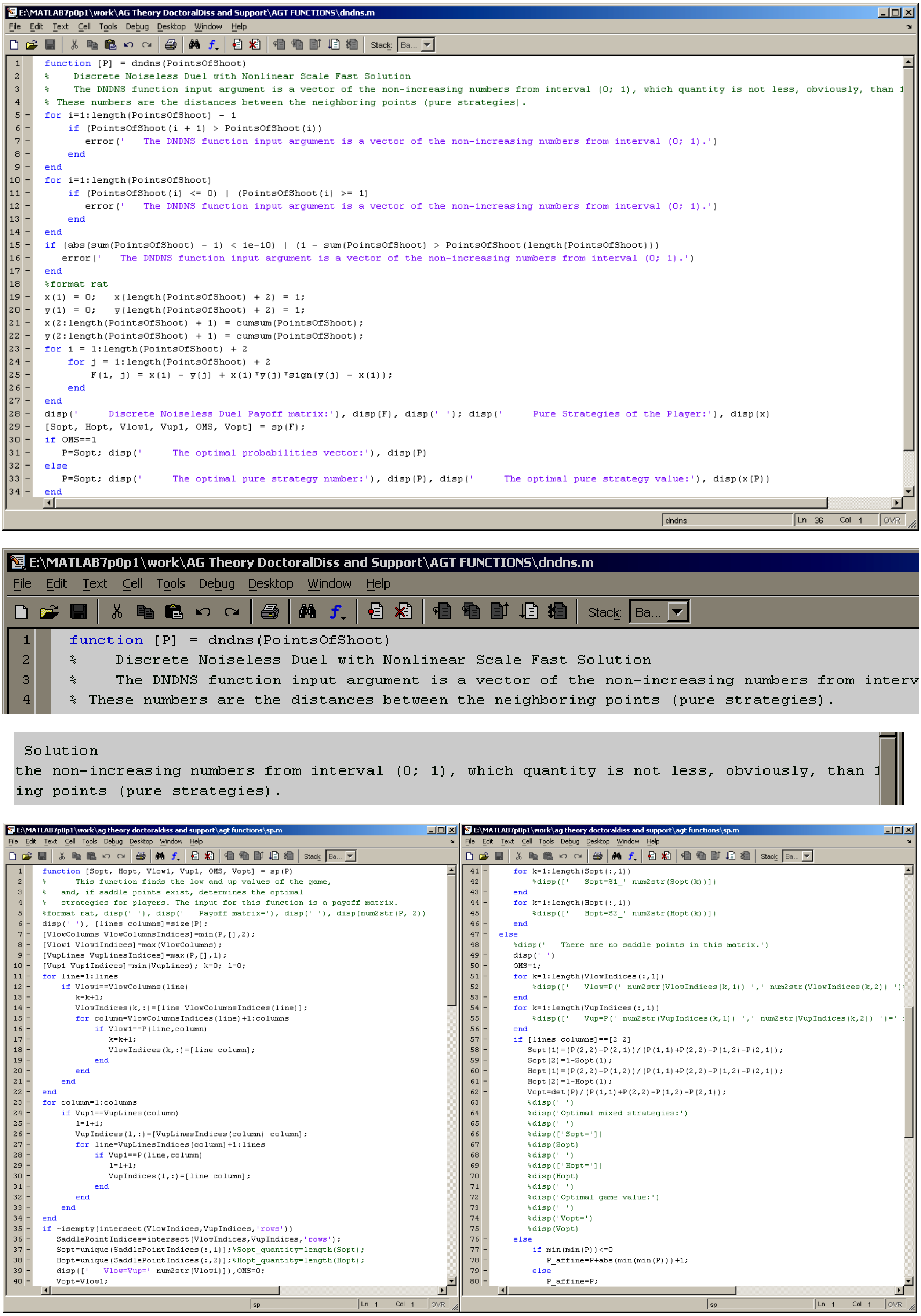
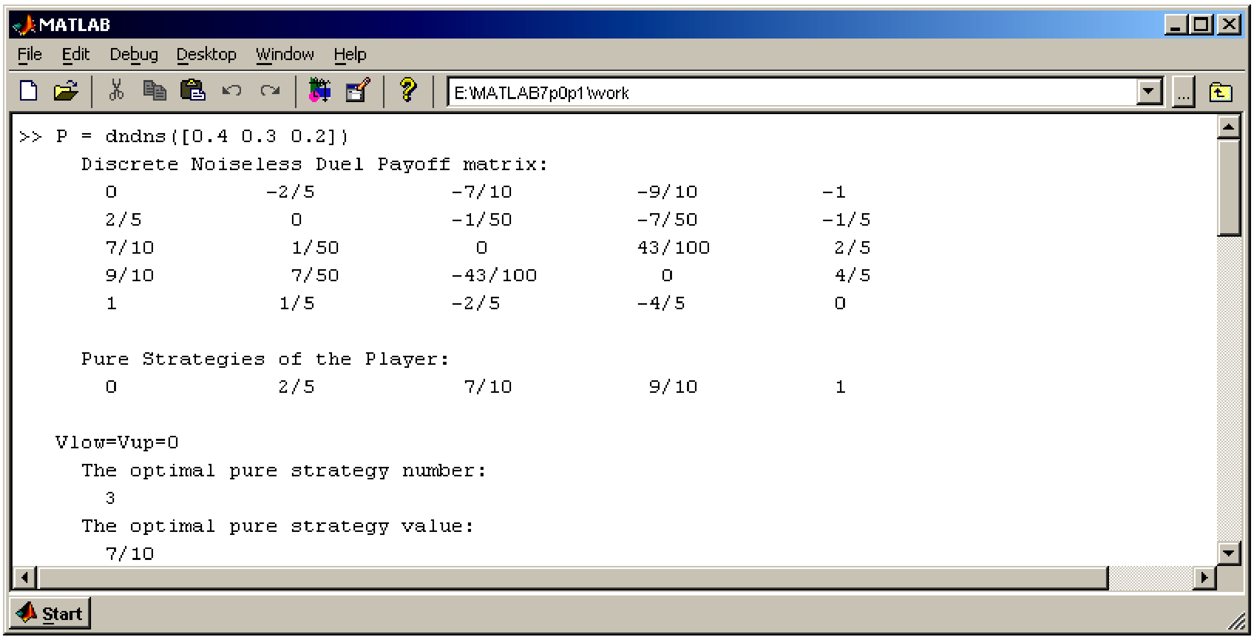


Figure 1. The MATLAB code of the module dndns for getting the fast solution [5] of discrete noiseless duel with the nonlinear scale on the linear accuracy functions



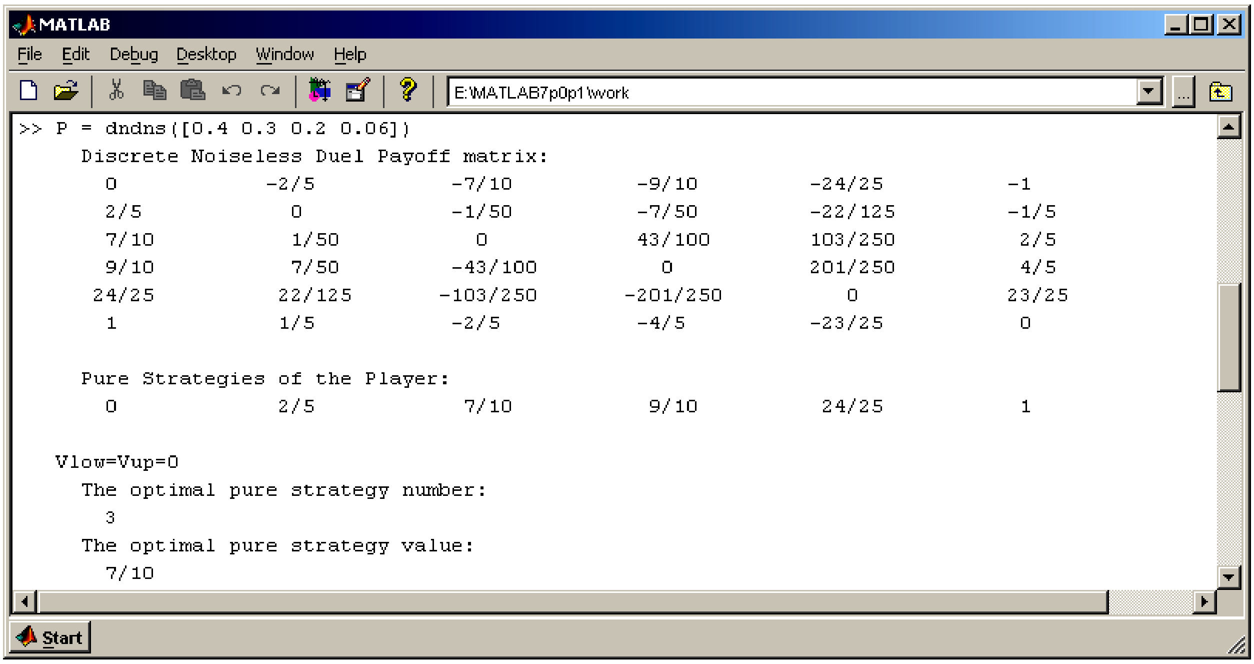
```
>> P = dndns([0.4 0.3 0.2])
   Discrete Noiseless Duel Payoff matrix:
     0           -2/5          -7/10         -9/10         -1
     2/5          0            -1/50         -7/50         -1/5
     7/10         1/50          0            43/100         2/5
     9/10         7/50         -43/100        0             4/5
     1            1/5          -2/5          -4/5           0

   Pure Strategies of the Player:
     0           2/5           7/10          9/10          1

 Vlow=Vup=0
   The optimal pure strategy number:
     3
   The optimal pure strategy value:
     7/10
```

Figure 2. The solution in the pure strategies $\{x_3, y_3\} = \left\{\frac{7}{10}, \frac{7}{10}\right\}$

for the $\left(f_{ij}\right)_{5\times5}$ **-game,**

consisting in the three distances between the neighboring points,

generating the pure strategies set $\left\{0, \frac{2}{5}, \frac{7}{10}, \frac{9}{10}, 1\right\}$ **of the player**



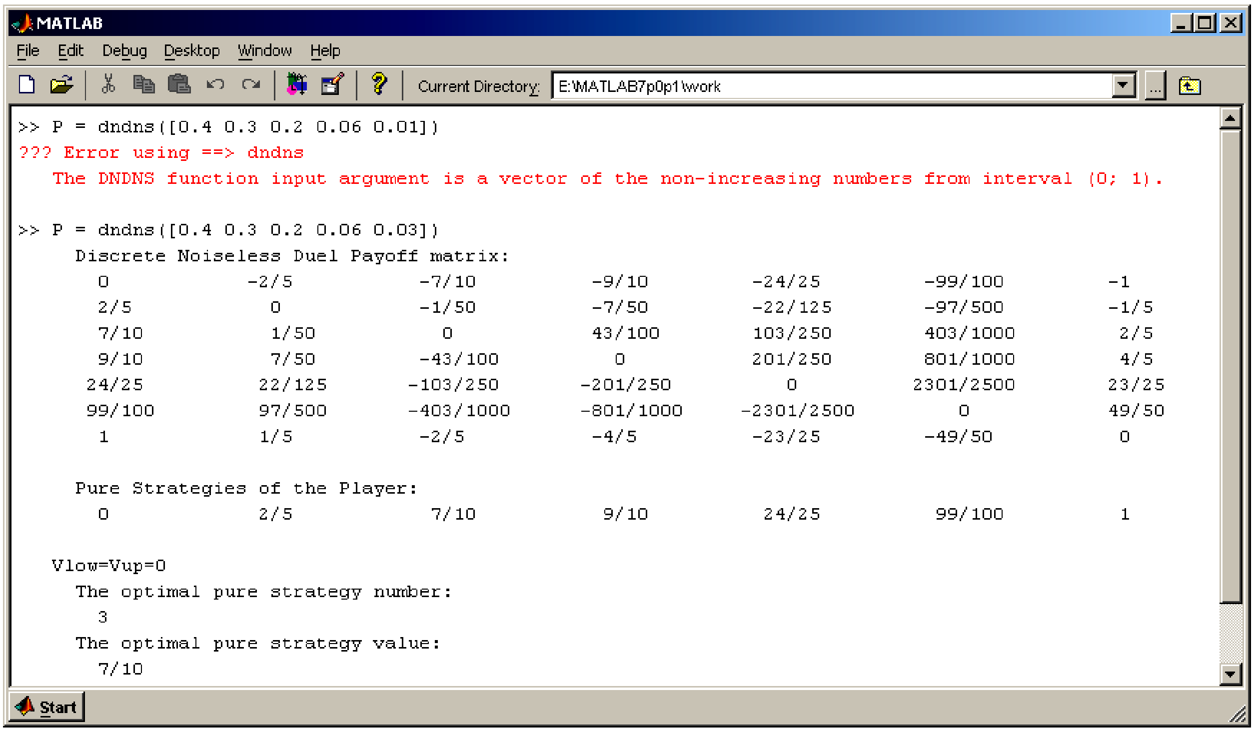
```
>> P = dndns([0.4 0.3 0.2 0.06])
   Discrete Noiseless Duel Payoff matrix:
     0           -2/5          -7/10         -9/10         -24/25        -1
     2/5          0            -1/50         -7/50         -22/125       -1/5
     7/10         1/50          0            43/100        103/250        2/5
     9/10         7/50         -43/100        0            201/250        4/5
    24/25        22/125        -103/250      -201/250        0            23/25
     1            1/5          -2/5          -4/5          -23/25         0

   Pure Strategies of the Player:
     0           2/5           7/10          9/10          24/25         1

 Vlow=Vup=0
   The optimal pure strategy number:
     3
   The optimal pure strategy value:
     7/10
```

Figure 3. The solution in the pure strategies $\{x_3, y_3\} = \left\{\frac{7}{10}, \frac{7}{10}\right\}$

for the $\left(f_{ij}\right)_{6\times6}$ **-game,**

consisting in the four distances between the neighboring points,

generating the pure strategies set $\left\{0, \frac{2}{5}, \frac{7}{10}, \frac{9}{10}, \frac{24}{25}, 1\right\}$ **of the player**

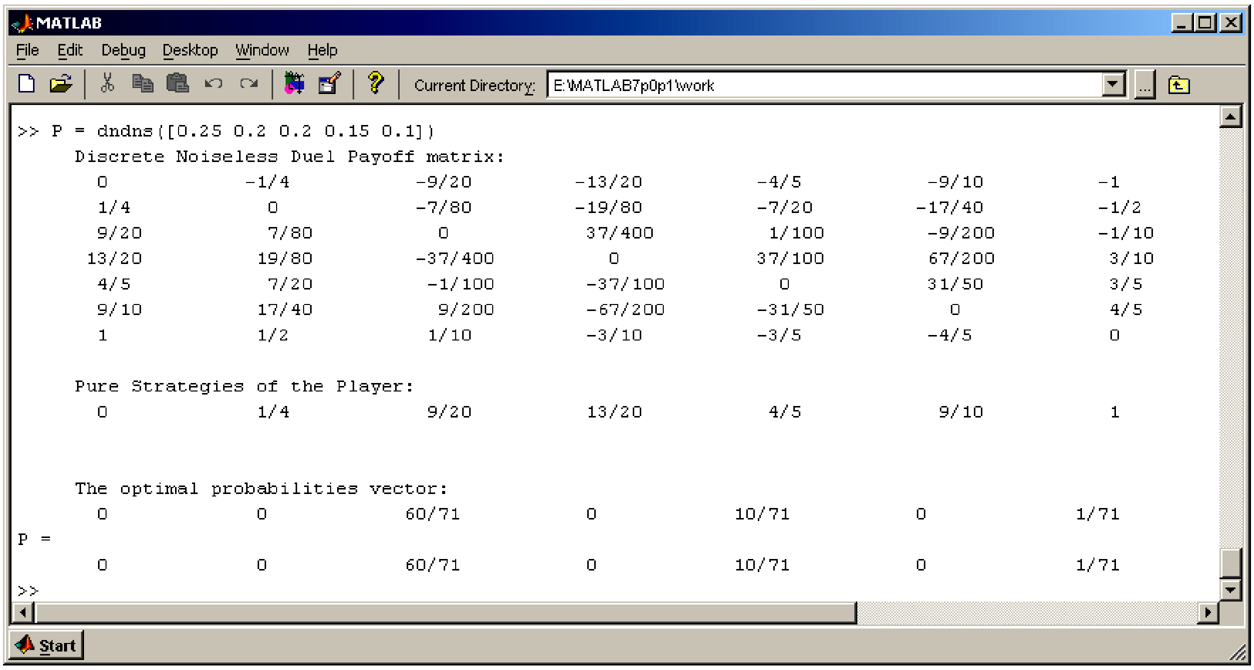



```
>> P = dndns([0.4 0.3 0.2 0.06 0.01])
??? Error using ==> dndns
   The DNDNS function input argument is a vector of the non-increasing numbers from interval (0; 1).

>> P = dndns([0.4 0.3 0.2 0.06 0.03])
     Discrete Noiseless Duel Payoff matrix:
       0          -2/5          -7/10          -9/10          -24/25          -99/100          -1
       2/5         0            -1/50          -7/50          -22/125         -97/500          -1/5
       7/10        1/50          0             43/100         103/250         403/1000          2/5
       9/10        7/50         -43/100          0            201/250         801/1000          4/5
      24/25       22/125       -103/250       -201/250          0            2301/2500         23/25
      99/100      97/500       -403/1000      -801/1000      -2301/2500         0              49/50
       1           1/5          -2/5           -4/5           -23/25          -49/50            0

     Pure Strategies of the Player:
       0           2/5           7/10           9/10           24/25           99/100           1

   Vlow=Vup=0
     The optimal pure strategy number:
       3
     The optimal pure strategy value:
       7/10
```

Figure 4. The erroneous inputting of the vector (3),

and the solution in the pure strategies $\{x_3,\ y_3\} = \left\{\frac{7}{10},\ \frac{7}{10}\right\}$

for the $\left(f_{ij}\right)_{7\times 7}$ **-game**




```
>> P = dndns([0.25 0.2 0.2 0.15 0.1])
     Discrete Noiseless Duel Payoff matrix:
       0          -1/4          -9/20         -13/20          -4/5          -9/10          -1
       1/4         0            -7/80         -19/80          -7/20        -17/40          -1/2
       9/20        7/80          0            37/400           1/100        -9/200         -1/10
      13/20       19/80        -37/400          0             37/100        67/200          3/10
       4/5         7/20         -1/100        -37/100           0           31/50           3/5
       9/10       17/40          9/200        -67/200         -31/50          0             4/5
       1           1/2           1/10          -3/10          -3/5          -4/5            0

     Pure Strategies of the Player:
       0           1/4           9/20          13/20           4/5           9/10           1


     The optimal probabilities vector:
       0           0            60/71            0            10/71            0            1/71
P =
       0           0            60/71            0            10/71            0            1/71
>>
```

Figure 5. The solution in the mixed strategies for another $\left(f_{ij}\right)_{7\times 7}$ **-game**

By the way, in the module `dndns` the solution is implied to be only the optimal strategy of the player, and this strategy, being pure or mixed, is returned into the assigned variable for further processing.

The conclusion and outlook for further programming investigation

The formulated discrete noiseless duel with the nonlinear scale on the linear accuracy functions has been conceived for modeling the conflict events, where in the course of time the player has more chances to shoot, that is the longer wait the more important pure strategy. The fast solution of this antagonistic game has been realized [6 — 11] thanking to the authorized MATLAB code `dndns`, using within also the authorized MATLAB code `sp` [5, 12]. The duel solution may be saved if needed and transferred to other math applications or databases for processing. The further programming investigation should be directed to the discrete noiseless duel sophistication, where there will be more than just the single bullet for the shot. Moreover, there must be explored the case with nonlinear accuracy functions $a_1(x)$ and $a_2(y)$, being, in general, the monotonous nondecreasing functions with the corresponding edge conditions, under which in the duel beginning the accuracy functions $a_1(0) = a_2(0) = 0$, and in the duel end the accuracy functions $a_1(1) = a_2(1) = 1$.

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