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MATHEMATICAL MODELING OF INTRUSION MOLDING

Abstract – The article is devoted to the mathematical modeling of intrusion forming of polymer products. intrusion moulding machine cycle involves the following stages. After the mould closes, the screw (while rotating) pushes forward to inject melt into the cooled mould. The air inside the mould will be pushed out through small vents at the furthest extremities of the melt flow path. When the cavity is filled, the screw continues to rotate to apply a holding pressure. This has the effect of squeezing extra melt into the cavity to compensate for the shrinkage of the plastic as it cools. This holding pressure is only effective as long as the gate remain open. Once the gate freeze, no more melt can enter the mould. When the molding has cooled to a temperature where it is solid enough to retain its shape, the mould opens and the molding is ejected. The mould then closes and the cycle is repeated. The study of the mathematical model makes it possible to determine melt output, depending on the elevation angle of the screw in the material cylinder based on the geometrical parameters of the extrusion head and the mold.

Keywords: polymer, molding, screw, intrusion.

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МАТМАТИЧНЕ МОДЕЛЮВАННЯ ПРОЦЕСУ ЛИТТЯ ПОЛІМЕРІВ ПІД ТИСКОМ

В статті розглянута математична модель процесу інтрузійного формування полімерного виробу. Прийнято до уваги, що розплавлений полімер заповнює оформлюючу порожнину прес-форми, при цьому повітря, що містилося у її середині, витискається через спеціальні отвори, а сам полімерний матеріал у прес-формі витримується деякий час під тиском для компенсації усадки готового виробу при охолодженні. Необхідний тиск у прес-формі забезпечується обертанням черв'яка литтєвого агрегату У результаті дослідження отриманої математичної моделі встановлені залежності витрати розплаву від кута підйому черв'яка у матеріальному циліндрі з урахуванням геометричних параметрів екструзійної головки та прес-форми.

Ключові слова: .полімер, формування, черв'як, інтрузія.

Introduction

One of the most common processing methods for plastics is intrusion molding. A typical intrusion moulding machine cycle involves the following stages. After the mould closes, the screw (while rotating) pushes forward to inject melt into the cooled mould. The air inside the mould will be pushed out through small vents at the furthest extremities of the melt flow path. When the cavity is filled, the screw continues to rotate to apply a holding pressure. This has the effect of squeezing extra melt into the cavity to compensate for the shrinkage of the plastic as it cools. This holding pressure is only effective as long as the gate remain open. Once the gate freeze, no more melt can enter the mould. When the molding has cooled to a temperature where it is solid enough to retain its shape, the mould opens and the molding is ejected. The mould then closes and the cycle is repeated.

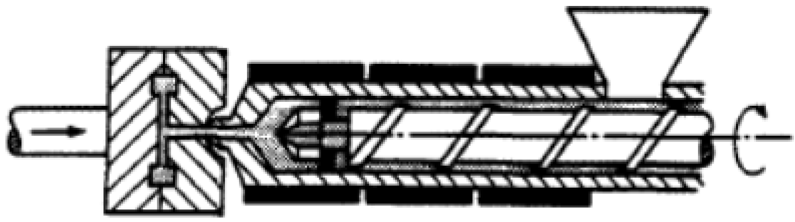

Fig. 1. Process of Intrusion Molding

Main content

Consider the flow of the melt between parallel plates as shown in Fig. 2(a).

For the small element of fluid ABCD the volume flow rate dQ is given by

$$dQ = V \cdot dy \cdot dx . \quad (1)$$

Assuming the velocity gradient is linear, then

$$V = V_d \cdot \left[\frac{y}{H} \right] .$$

Substituting in (1) and integrating over the channel depth, H, then the total drag flow, Qd, is given by

$$Q_d = \int_0^T \int_0^H \frac{V_d \cdot y}{H} \cdot dy \cdot dx ;$$

$$Q_d = \frac{1}{2} \cdot T \cdot H \cdot V_d . \quad (2)$$

This may be compared to the situation in the extruder where the fluid is being dragged along by the relative movement of the screw and barrel. Fig. 1.3 shows the position of the element of fluid and (2) may be modified to include terms relevant to the extruder dimensions.

For example

$$V_d = \pi \cdot D \cdot N \cdot \cos\varphi ,$$

where N is the screw speed (in revolutions per unit time).

$$T = (\pi \cdot D \cdot \tan\varphi - e) \cdot \cos\varphi ;$$

$$Q_d = \frac{1}{2} \cdot (\pi \cdot D \cdot \tan\varphi - e) \cdot (\pi \cdot D \cdot N \cdot \cos^2 \varphi) \cdot H .$$

In most cases the term, e, is small in comparison with $\pi \cdot D \cdot \tan\varphi$ so this expression is reduced to

$$Q_d = \frac{1}{2} \cdot \pi^2 \cdot D^2 \cdot N \cdot H \cdot \sin\varphi \cdot \cos\varphi . \quad (3)$$

Note that the shear rate in the metering zone will be given by Vd/H.

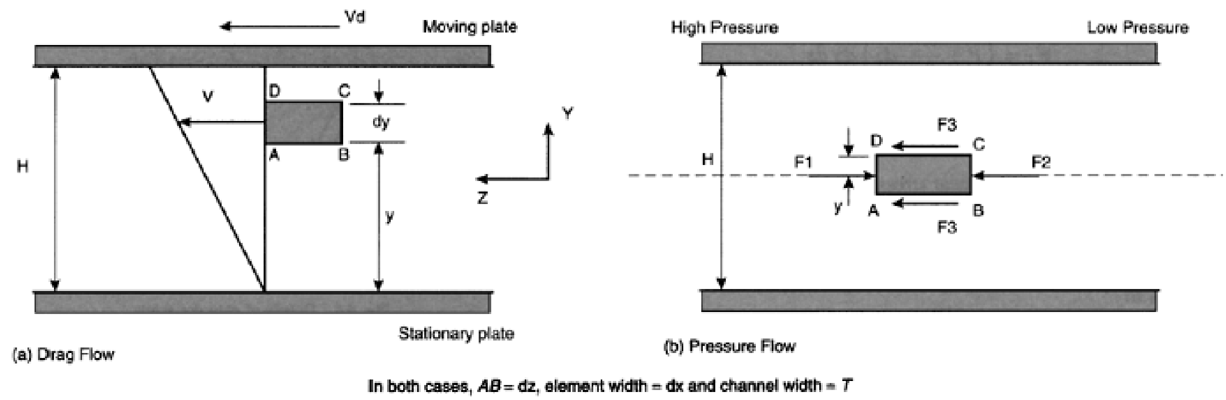

Fig. 2. Melt Flow between parallel plates

Consider the element of fluid shown in Fig. 2(b). The forces are

$$F_1 = \left(P + \frac{\partial P}{\partial z} \cdot dz \right) \cdot dy \cdot dx ;$$

$$F_2 = P \cdot dy \cdot dx ;$$

$$F_3 = \tau_y \cdot dz \cdot dx ,$$

where P is pressure and τ is the shear stress acting on the element. For steady flow these forces are in equilibrium so they may be equated as follows:

$$F_1 = F_2 + 2 \cdot F_3 ,$$

which reduces to

$$y \cdot \frac{dP}{dz} = \tau_y . \tag{4}$$

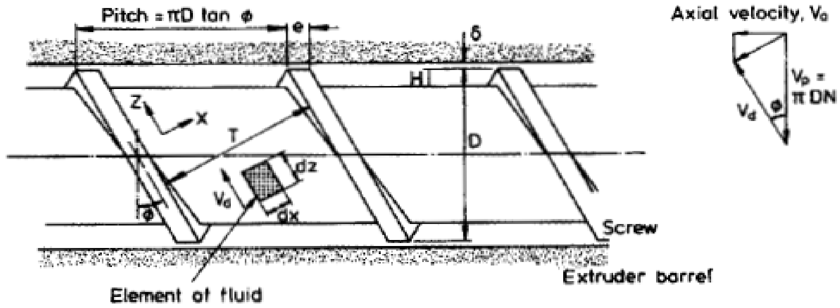

Fig. 3. Details of screw

Now for a Newtonian fluid, the shear stress, τ_y, is related to the viscosity, η, and the shear rate, $\dot{\gamma}$, by the equation

$$\tau_y = \eta \cdot \dot{\gamma} = \eta \cdot \frac{dV}{dy} .$$

Using this in equation (4)

$$y \cdot \frac{dP}{dz} = \eta \cdot \frac{dV}{dy}$$

Integrating

$$\int_0^V dV = \frac{1}{\eta} \cdot \frac{dP}{dz} \cdot \int_{\frac{H}{2}}^{y} y dy$$

So

$$V = \frac{1}{\eta} \cdot \frac{dP}{dz} \cdot \left(\frac{y^2}{2} - \frac{H^2}{8} \right) . \tag{5}$$

Also, for the element of fluid of depth, *dy*, at distance, *y*, from the centre line (and whose velocity is *V*) the elemental flow rate, *dQ*, is given by

$dQ = V \cdot T \cdot dy$.

This may be integrated to give the pressure flow, *Qp*

$$Q_p = 2 \cdot \int_0^{\frac{H}{2}} \frac{1}{\eta} \cdot \frac{dP}{dz} \cdot T \cdot \left(\frac{y^2}{2} - \frac{H^2}{8} \right) \cdot dy ;$$

$$Q_p = -\frac{1}{12 \cdot \eta} \cdot \frac{dP}{dz} \cdot T \cdot H^3 . \tag{6}$$

Referring to the element of fluid between the screw flights as shown in Fig. 3, this equation may be rearranged using the following substitutions.

Assuming e is small,

$$T = \pi \cdot D \cdot \tan \varphi \cdot \cos \varphi .$$

Also,

$$\sin \varphi = \frac{dL}{dz}$$

so

$$\frac{dP}{dz} = \frac{dP}{dL} \cdot \sin \varphi .$$

Thus the expression for *Qp* becomes

$$Q_p = \frac{\pi \cdot D \cdot H^3 \cdot \sin^2 \varphi}{12 \cdot \eta} \cdot \frac{dP}{dL} . \tag{7}$$

For many practical purposes sufficient accuracy is obtained by neglecting the leakage flow term. In addition the pressure gradient is often considered as linear so

$$\frac{dP}{dL}=\frac{P}{L},$$

where 'L' is the length of the extruder. In practice the length of an extruder screw can vary between 17 and 30 times the diameter of the barrel.

The total output is the combination of drag flow and back pressure flow:

$$Q=\frac{1}{2}\cdot\pi^2\cdot D^2\cdot N\cdot H\cdot\sin\varphi\cdot\cos\varphi-\frac{\pi\cdot D\cdot H^3\cdot\sin^2\varphi}{12\cdot\eta}\cdot\frac{P}{L}. \tag{8}$$

From equation (8) it may be seen that there are two interesting situations to consider. One is the case of free discharge where there is no pressure build up at the end of the extruder so

$$Q=\frac{1}{2}\cdot\pi^2\cdot D^2\cdot N\cdot H\cdot\sin\varphi\cdot\cos\varphi. \tag{9}$$

The other case is where the pressure at the end of the extruder is large enough to stop the output. From (8) with $Q = 0$ and ignoring the leakage flow

$$P=P_{\max}=\frac{6\cdot\pi\cdot D\cdot L\cdot N\cdot\eta}{H^2\cdot\tan\varphi}. \tag{10}$$

It is interesting to note that when a die and a mold are coupled to the extruder their requirements are conflicting. The extruder has a high output if the pressure at its outlet is low. However, the outlet from the extruder is the inlet to the die and the output of the latter increases with inlet pressure.

The output, Q, of a Newtonian fluid from a tube is given by a relation of the form

$$Q=K\cdot\Delta P, \tag{11}$$

where $K=\frac{\pi\cdot R^4}{8\cdot\eta\cdot L_d}$ for a capillary tube of radius R and length Ld.

For a die

$$Q=\frac{\pi\cdot {R_d}^4}{8\cdot\eta\cdot L_d}\cdot\left(P-P_d\right). \tag{12}$$

For a mold

$$Q=\frac{\pi\cdot {R_m}^4}{8\cdot\eta\cdot L_m}\cdot\left(P_d-P_m\right). \tag{13}$$

Assuming that during the filling process $P_m=0$ from (11) and (12):

$$\frac{{R_d}^4}{L_d}\cdot\left(P-P_d\right)=\frac{{R_m}^4}{L_m}\cdot P_d \tag{14}$$

and

$$P_d=\frac{\dfrac{{R_d}^4}{L_d}}{\dfrac{{R_m}^4}{L_m}+\dfrac{{R_d}^4}{L_d}}\cdot P=\frac{1}{1+\dfrac{{R_m}^4}{{R_d}^4}\cdot\dfrac{L_d}{L_m}}\cdot P. \tag{15}$$

Then (12) and (13) will be:

$$Q=\frac{\pi\cdot {R_d}^4}{8\cdot\eta\cdot L_d}\cdot\left(1-\frac{1}{1+\dfrac{{R_m}^4}{{R_d}^4}\cdot\dfrac{L_d}{L_m}}\right)\cdot P. \tag{16}$$

$$Q=\frac{\pi\cdot {R_m}^4}{8\cdot\eta\cdot L_m}\cdot\frac{1}{1+\dfrac{{R_m}^4}{{R_d}^4}\cdot\dfrac{L_d}{L_m}}\cdot P. \tag{17}$$

Substituting (17) in (8) we obtain:

$$\frac{1}{2}\cdot\pi^2\cdot D^2\cdot N\cdot H\cdot\sin\varphi\cdot\cos\varphi-$$
$$-\frac{\pi\cdot D\cdot H^3\cdot\sin^2\varphi}{12\cdot\eta}\cdot\frac{P}{L}=\frac{\pi\cdot {R_m}^4}{8\cdot\eta\cdot L_m}\cdot\frac{1}{1+\dfrac{{R_m}^4}{{R_d}^4}\cdot\dfrac{L_d}{L_m}}\cdot P\cdot \tag{18}$$

From which the pressure at the operating point:

$$P = \frac{\frac{1}{2} \cdot \pi \cdot D^2 \cdot N \cdot H \cdot \sin\varphi \cdot \cos\varphi}{\frac{R_m^{\ 4}}{8 \cdot \eta \cdot L_m} \cdot \frac{1}{1 + \frac{R_m^{\ 4}}{R_d^{\ 4}} \cdot \frac{L_d}{L_m}} + \frac{D \cdot H^3 \cdot \sin^2\varphi}{12 \cdot \eta \cdot L}}. \tag{19}$$

Substitution (19) in (8) gives the output:

$$Q = \frac{1}{2} \cdot \pi^2 \cdot D^2 \cdot N \cdot H \cdot \sin\varphi \cdot \cos\varphi -$$

$$- \frac{\pi \cdot D \cdot H^3 \cdot \sin^2\varphi}{12 \cdot \eta \cdot L} \cdot \frac{\frac{1}{2} \cdot \pi \cdot D^2 \cdot N \cdot H \cdot \sin\varphi \cdot \cos\varphi}{\frac{R_m^{\ 4}}{8 \cdot \eta \cdot L_m} \cdot \frac{1}{1 + \frac{R_m^{\ 4}}{R_d^{\ 4}} \cdot \frac{L_d}{L_m}} + \frac{D \cdot H^3 \cdot \sin^2\varphi}{12 \cdot \eta \cdot L}}. \tag{20}$$

In Fig. 4 the dependence of pressure at the operating point on the a screw flight angle is represented for the following set of characteristics: screw diameter = 40 mm; flight depth = 3 mm; L/D ratio = 24; screw speed = 100 rev/min; melt viscosity of 50 Ns/m2; die channel radius 5 mm; mold channel radius 5 mm; die channel length 10 mm; mold channel length 10 mm.

In Fig. 5 the dependence of melt output on the screw flight angle is represented.

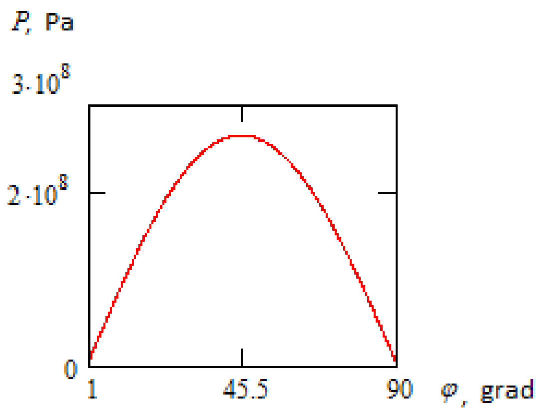

Fig. 4.Tthe dependence of pressure at the operating point on the a screw flight angle (Eq. (19))

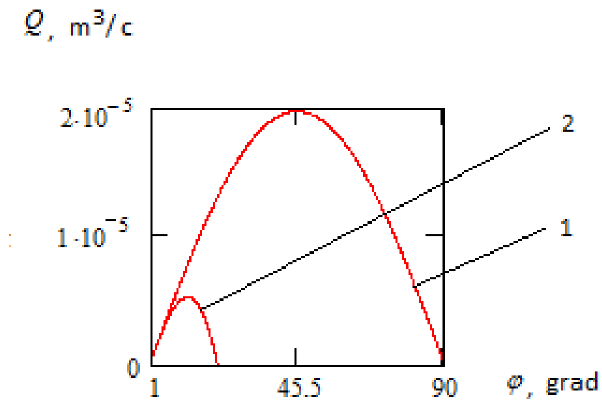

Fig. 5. The dependence of the melt output on the screw flight angle: 1 - free discharge (Eq. (9)); 2 – intrusion molding (Eq. (20))

Summary

The maximum output would be obtained if the screw flight angle was about 10°. In practice a screw flight angle of 17.7° is frequently used because this is the angle which occurs if the pitch of the screw is equal to the diameter and so it is convenient to manufacture and for a considerable portion of the extruder length, the screw is acting as a solids conveying device and it is known that the optimum angle in such cases is 17° to 20°.

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