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O.A. PASTUKH
Ternopil Ivan Pul'uj National Technical University

MATHEMATICAL MODEL OF INFORMATION TECHNOLOGY OF TRANSFER FUZZY INFORMATION BY QUANTUM COMMUNICATION CHANNEL

Abstract –The mathematical model of information technology of transfer fuzzy information by quantum communication channel has been developed in the article. It provides absolute protection of transfer fuzzy information. The basically absolute protection is quantum teleportation. Fuzzy information is fuzzy variable. Mathematical description of fuzzy variables is membership function. Quantum information are quantum fuzzy variables. Mathematical description of quantum fuzzy variables is wave function. Mathematical models of transform fuzzy information to quantum fuzzy information are unitary matrix. Unitary matrix are noncommutative multiplicative group. Quantum communications are multiplicative unitary matrix on the wave function. Communications quantum channel are noncommutative group unitary operators. Mathematical description of quantum teleportation are not presentation tensor multiplication unitary operators.

Keywords: mathematical model, information technology, quantum teleportation.

О.А. ПАСТУХ
Тернопільський національний технічний університет імені І.Пулюя

МАТЕМАТИЧНЕ ЗАБЕЗПЕЧЕННЯ ІНФОРМАЦІЙНОЇ ТЕХНОЛОГІЇ ПЕРЕДАЧІ НЕЧІТКОЇ ІНФОРМАЦІЇ ПО КВАНТОВОМУ КАНАЛУ ЗВ'ЯЗКУ

У статті запропоновано математичне забезпечення інформаційної технології передачі нечіткої інформації по квантовому каналу зв'язку за допомогою квантової телепортації.

Ключові слова: математичне забезпечення, інформаційна технологія, квантова телепортація.

Вступ

Constant growth of information volumes causes requirements to use new technologies for transmission over computer networks. While there are requirements for speed and reliability of information transmission. Such requirements are made in particular to the specific type of information – fuzzy information.

Among the new information technologies of information transfer, which are developing rapidly, there are quantum information technologies.

Because of quantum teleportation has almost a commercial level in quantum information technologies of information transfer, it is logical to try to use it for the transfer fuzzy information through a quantum communication channel. This idea is interesting for the reason that quantum teleportation provides absolute protection of transferred information.

But quantum teleportation in particular, as every information technology, is based on some mathematical providing. Today the development of mathematical software of information technology of fuzzy information transfer through a quantum communication channel with help of quantum teleportation is a separate scientific problem. This article discovers such scientific question.

Quantum teleportation as information transmission method was first considered by Einstein, Podolsky and Rosen. However, for the first time quantum teleportation experimentally has been implemented by the Austrian group of scientists headed by Zeilinger in 1998. Later experimental implementation of quantum teleportation was carried out in different years by different scientific groups of academic institutions around the world.

The aim of the article is to offer mathematical software for information technology of fuzzy information transfer.

Statement of the problem is to develop mathematical software of information technology of fuzzy information transfer through a quantum communication channel.

The main part

An important component of information technology for information transfer is mathematical software. The mathematical software generates the potential for effective and reliable information transmission through communication channels. All this applies to the case of fuzzy information, if it is passed through a quantum communication channel.

Various quantum information technologies are used for information transmission through a quantum communication channel. Among these quantum information technologies there is a quantum teleportation.

In order to fully use the features of quantum teleportation as information technology for the transmission of fuzzy information through a quantum communication channel, we need to develop software that uses a particular type of quantum information named quantum vague information. A mathematical representation of quantum fuzzy information is given in the work [1]. [1] shows that the quantum fuzzy information is presented by quantum fuzzy information qfa that uniquely and fully is specified by wave function ψ_{qfa}.

Let fuzzy information is presented by fuzzy data fa that clearly and fully is defined by its membership function $I_{fa}(x)$.

Coding U $_{fa\to qfa}$ of fuzzy data fa in quantum fuzzy data qfa can be performed by the consecutive actions of the operators H and U (that is, U $_{fa\to qfa}$ = $\mathbf{U}\cdot\mathbf{H}$), which matrices have according type:

$$\mathbf{H}=\begin{pmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ 1/\sqrt{2} & -1/\sqrt{2}\end{pmatrix},\ \mathbf{U}=\begin{pmatrix}I_{fa}(x_1) & I_{fa}(x_2)\\ I_{fa}(x_2) & -I_{fa}(x_1)\end{pmatrix}.$$

Thus, acting on an initialized state of the quantum information vehicle (quantum bit) by operator U $_{fa\to qfa}$ its state conforms the quantum fuzzy data qfa that is already possible to be teleported directly. Teleportation of quantum fuzzy data qfa , that is contained in quantum bit, is followed in the way. First, quantum fuzzy information is denoted as $\left|\psi_{qfa}\right\rangle = I_{fa}(x_1)e^{i\varphi_\alpha}\left|0\right\rangle + I_{fa}(x_2)e^{i\varphi_\beta}\left|1\right\rangle$. Let there is tangled pair of quantum bits in a Bell state, that is $\left|\psi_{bell\ \ state}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|00\right\rangle + \left|11\right\rangle\right)$.

One of tangled quantum bits is the point of departure of quantum fuzzy information $\left|\psi_{qfa}\right\rangle$, and the second is in its reception.

General state of system – a quantum bit in the status $\left|\psi_{qfa}\right\rangle$and tangled pair of quantum bits in a state $\left|\psi_{bell\ \ state}\right\rangle$ – is:

$$\left|\psi_0\right\rangle = \left|\psi_{qfa}\right\rangle \otimes \left|\psi_{bell\ \ state}\right\rangle = \frac{1}{\sqrt{2}}\left(I_{fa}(x_1)e^{i\varphi_\alpha}\left|0\right\rangle\left(\left|00\right\rangle + \left|11\right\rangle\right) + I_{fa}(x_2)e^{i\varphi_\beta}\left|1\right\rangle\left(\left|00\right\rangle + \left|11\right\rangle\right)\right),$$

where $\otimes$ – the tensor product.

Then we follow actions that implement the process of teleportation. Acting by operator CNOT on the state ψ_0, it becomes a state $\left|\psi_1\right\rangle = \frac{1}{\sqrt{2}}\left(I_{fa}(x_1)e^{i\varphi_\alpha}\left|0\right\rangle\left(\left|00\right\rangle + \left|11\right\rangle\right) + I_{fa}(x_2)e^{i\varphi_\beta}\left|1\right\rangle\left(\left|10\right\rangle + \left|01\right\rangle\right)\right)$, if quantum bit in a state $\left|\psi_{qfa}\right\rangle$ is the managing and quantum bit of tangled pair in the point of departure of quantum fuzzy information is managed.

Then, acting as the operator H on the quantum bit in a state $\left|\psi_{qfa}\right\rangle$, $\left|\psi_1\right\rangle$ turns to the state:

$$\left|\psi_2\right\rangle = \frac{1}{2}\left(I_{fa}(x_1)e^{i\varphi_\alpha}\left(\left|0\right\rangle + \left|1\right\rangle\right)\left(\left|00\right\rangle + \left|11\right\rangle\right) + I_{fa}(x_2)e^{i\varphi_\beta}\left(\left|0\right\rangle - \left|1\right\rangle\right)\left(\left|10\right\rangle + \left|01\right\rangle\right)\right) =$$

$$= \frac{1}{2}\left(\left|00\right\rangle\left(I_{fa}(x_1)e^{i\varphi_\alpha}\left|0\right\rangle + I_{fa}(x_2)e^{i\varphi_\beta}\left|1\right\rangle\right) + \left|01\right\rangle\left(I_{fa}(x_2)e^{i\varphi_\beta}\left|0\right\rangle + I_{fa}(x_1)e^{i\varphi_\alpha}\left|1\right\rangle\right) +\right.$$

$$\left. + \left|10\right\rangle\left(I_{fa}(x_1)e^{i\varphi_\alpha}\left|0\right\rangle - I_{fa}(x_2)e^{i\varphi_\beta}\left|1\right\rangle\right) + \left|11\right\rangle\left(-I_{fa}(x_2)e^{i\varphi_\beta}\left|0\right\rangle + I_{fa}(x_1)e^{i\varphi_\alpha}\left|1\right\rangle\right)\right).$$

$\left|\psi_2\right\rangle$ shows that the quantum bit of tangled pair in reception of quantum fuzzy information is located in one of the states:

$$\left|\tilde{\psi}_1\right\rangle = I_{fa}(x_1)e^{i\varphi_\alpha}\left|0\right\rangle + I_{fa}(x_2)e^{i\varphi_\beta}\left|1\right\rangle,\qquad \left|\tilde{\psi}_2\right\rangle = I_{fa}(x_2)e^{i\varphi_\beta}\left|0\right\rangle + I_{fa}(x_1)e^{i\varphi_\alpha}\left|1\right\rangle,$$

$$\left|\tilde{\psi}_3\right\rangle = I_{fa}(x_1)e^{i\varphi_\alpha}\left|0\right\rangle - I_{fa}(x_2)e^{i\varphi_\beta}\left|1\right\rangle,\qquad \left|\tilde{\psi}_4\right\rangle = -I_{fa}(x_2)e^{i\varphi_\beta}\left|0\right\rangle + I_{fa}(x_1)e^{i\varphi_\alpha}\left|1\right\rangle$$

each of which must be turned into quantum fuzzy information transmitted $\left|\psi_{qfa}\right\rangle$

Then the result of measuring of the state of the quantum bits in the point of departure of quantum fuzzy information is transmitted to the receiving point for quantum fuzzy information through classical communication channel.

If the dimension has given a result $\left|00\right\rangle$, the status $\left|\tilde{\psi}_1\right\rangle$ of a quantum bit of tangled pair in reception of quantum fuzzy information corresponds teleported quantum state $\left|\psi_{qfa}\right\rangle$.

If $-\left|01\right\rangle$, then Pauli operator X affects on a state $\left|\tilde{\psi}_2\right\rangle$ of quantum bit of tangled pair in reception of quantum fuzzy information, which matrix is following

$$\mathbf{X}=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

After that, its status matches teleported status:

$$\mathbf{X}\left|\tilde{\psi}_2\right\rangle = \mathbf{X}\left(I_{fa}(x_2)e^{i\varphi_\beta}\left|0\right\rangle + I_{fa}(x_1)e^{i\varphi_\alpha}\left|1\right\rangle\right) = I_{fa}(x_1)e^{i\varphi_\alpha}\left|0\right\rangle + I_{fa}(x_2)e^{i\varphi_\beta}\left|1\right\rangle = \left|\psi_{qfa}\right\rangle.$$

If $-|10\rangle$, then operator Pauli Z affects on the state $|\tilde{\psi}_3\rangle$, which matrix has the form:

$$\mathbf{Z}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}.$$

Therefore, the status matches teleported status:

$$\mathbf{Z}|\tilde{\psi}_3\rangle=\mathbf{Z}\left(I_{fa}(x_1)e^{i\varphi_\alpha}|0\rangle-I_{fa}(x_2)e^{i\varphi_\beta}|1\rangle\right)=I_{fa}(x_1)e^{i\varphi_\alpha}|0\rangle+I_{fa}(x_2)e^{i\varphi_\beta}|1\rangle=|\psi_{qfa}\rangle.$$

If $-|11\rangle$, then Pauli operators X and Z affect consistently on the status $|\tilde{\psi}_4\rangle$. In consequence of this, the state of quantum bit of tangled pair in reception of quantum fuzzy information corresponds teleported status:

$$(\mathbf{Z}\circ\mathbf{X})|\tilde{\psi}_4\rangle=(\mathbf{Z}\circ\mathbf{X})\left(-I_{fa}(x_2)e^{i\varphi_\beta}|0\rangle+I_{fa}(x_1)e^{i\varphi_\alpha}|1\rangle\right)=I_{fa}(x_1)e^{i\varphi_\alpha}|0\rangle+I_{fa}(x_2)e^{i\varphi_\beta}|1\rangle=|\psi_{qfa}\rangle.$$

where o– is the operation of composition of operators.

Thus, in the reception there is a quantum bit, the status of which corresponds to a quantum fuzzy information that is transmitted $|\psi_{qfa}\rangle$.

This means that quantum fuzzy information through a quantum communication channel has been transferred with help of quantum teleportation.

Then decoding of quantum fuzzy data in fuzzy data *qfa* into the fuzzy data *fa* that is inverse transformation of quantum fuzzy information in vague information.

The above results are mathematical software of information technology of fuzzy information transfer through a quantum communication channel with help quantum teleportation, on the basis of quantum fuzzy information.

Conclusions

The mathematical software of information technology of fuzzy information transfer through a quantum communication channel with the help of quantum teleportation has been developed in the article that allows to organize reliable protection

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Література

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