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PURE STRATEGIES PARETO EFFICIENT SITUATIONS VERSUS PURE STRATEGIES NASH EQUILIBRIUM SITUATIONS BY THEIR STOCHASTICALLY CONSTRAINED PAYOFFS IN DYADIC GAME MODELING OF RESOURCES RATIONAL USAGE WITH ALTERNATIVE CHOICE OF ACTION

There is considered contrariety of Pareto efficient situations and Nash equilibrium situations in dyadic games. Their players' payoffs are constrained by normal and uniform laws. Solving these games in pure strategies, there is the advantageousness of maximal mean payoffs in Pareto efficient situations. In resources rational usage, this fact can be applied for projecting collective work at more efficient rate.

Keywords: resources rational usage, alternative choice of action, dyadic games, pure strategy, Nash equilibrium, Pareto efficiency.

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ПАРЕТО-ЕФЕКТИВНІ СИТУАЦІЇ У ЧИСТИХ СТРАТЕГІЯХ ПРОТИ РІВНОВАЖНИХ ЗА НЕШЕМ СИТУАЦІЙ У ЧИСТИХ СТРАТЕГІЯХ ЗА СТОХАСТИЧНО ОБМЕЖУВАНИХ ВИГРАШІВ У ДІАДИЧНОМУ ІГРОВОМУ МОДЕЛЮВАННІ РАЦІОНАЛЬНОГО ВИКОРИСТАННЯ РЕСУРСІВ З АЛЬТЕРНАТИВНИМ ВИБОРОМ ДІЇ

Розглядається розбіжність між ситуаціями, що є ефективними за Парето, і рівноважними ситуаціями за Нешем у діадичних іграх. Виграші їх гравців обмежуються за нормальним та рівномірними законами. Розв'язуючи ці ігри у чистих стратегіях, з'являється вигідність максимальних середніх виграшів у ситуаціях, що є ефективними за Парето. У раціональному використанні ресурсів цей факт може бути застосований для планування колективної роботи на більш ефективному рівні.

Ключові слова: раціональне використання ресурсів, альтернативний вибір дії, діадичні ігри, чиста стратегія, рівновага за Нешем, ефективність за Парето.

Resources rational usage with alternative choice of action by dyadic games

Alternative choice of action occurs in many technical and industrial branches. This is an evidence of narrowing the choice range, no matter how wide it may be, into the binary mode. Binarization helps to rationalize decisions faster. However, real aftermath of even the dual choice is always uncertain, because it is influenced with choices of other participants of the interaction process. The uncertainty is unwanted, but it is unavoidable. For instance, in unsupervised frequency allocation, a superfluous amount of network-connected users may temporarily disconnect a part of them. This is also denial of service, when server runs out of resources while number of queries is not less than the rejection number. In machinery, wrong estimation of run-in period may cause either underuse or overuse of the tool, mechanism, engine, etc. The same concerns the manufacturing resources whose rational usage depends on enablement (power control) of separate units. To work optimally, there are noncooperative games for modeling the interaction of the participants (users) or players, having own payoffs in any situation [1, 2]. For alternative choice of action, these games are dyadic. And their solutions are nonetheless uncertain as there are several types of them, and not always desirable symmetrical situation is the most advantageous.

Problem of taking the most advantageous pure strategy

In dyadic $\mathscr{F}$ -game

$$\left\langle \left\{ D_i \right\}_{i=1}^{n}, \left\{ \mathbf{P}_i = \left[p_J^{\langle i \rangle} \right]_{\mathscr{F}} \right\}_{i=1}^{n} \right\rangle \tag{1}$$

of $n \in \mathbb{N} \setminus \{1\}$ players by $\mathscr{F} = \bigtimes_{k=1}^{n} 2$, the i -th player has the set $D_i = \{x_i, y_i\}$ of its pure strategies and n -dimensional $\mathscr{F}$ -matrix $\mathbf{P}_i = \left[p_J^{\langle i \rangle} \right]_{\mathscr{F}}$ of its payoffs by

$$J = \left\{ j_k \right\}_{k=1}^{n} = \left\{ j_k : j_k \in \{1, 2\} \; \forall k = \overline{1, n} \right\}, \tag{2}$$

where in the situation $\left\{ z_k \right\}_{k=1}^{n} \in \bigtimes_{k=1}^{n} D_k$ denoting $z_k \in \{x_k, y_k\}$ the i -th player gets the payoff $p_J^{\langle i \rangle}$ at $j_i = 1$ for $z_i = x_i$ and $j_i = 2$ for $z_i = y_i$. For practicing at short terms, only situations in pure strategies being the most advantageous are sought. Popular types of advantageousness are Nash equilibrium and Pareto efficiency [3]. Thus, situation $\left\{ z_k^* \right\}_{k=1}^{n} \in \bigtimes_{k=1}^{n} D_k$ is a pure strategies Nash equilibrium situation (PSNES) if $p_{J_*}^{\langle i \rangle} \geqslant p_{J_i}^{\langle i \rangle} \;\; \forall i = \overline{1, n}$ by the set

J_* corresponding to $\{z_k^*\}_{k=1}^n$ and the set J_i corresponding to $\left\{\{z_k^*\}_{k=1}^n \setminus \{z_i^*\}\right\} \bigcup \{z_i\}$. And if the players have a treaty on applying strategies from PSNES, then a player's individual avoidance of its strategy from PSNES is disadvantageous. Nevertheless, game may have a few PSNES, bringing various payoffs for the player. And the Nash equilibrium does not imply profitability. Nor implies the equity, unless a PSNES is symmetrical. To some contrary, situation $\{\hat{z}_k\}_{k=1}^n \in \bigtimes_{k=1}^n D_k$ is a pure strategies Pareto efficient situation (PSPES) if there is no situation $\{z_k\}_{k=1}^n$ that $p_J^{\langle i \rangle} \geqslant p_{\hat{J}}^{\langle i \rangle}$ $\forall i = \overline{1, n}$ and $\exists\, l \in \left\{\overline{1, n}\right\}$ with $p_J^{\langle l \rangle} > p_{\hat{J}}^{\langle l \rangle}$ by the set $\hat{J}$ corresponding to $\{\hat{z}_k\}_{k=1}^n$. But in some PSPES a player avoiding its Pareto efficient strategy can get greater payoff. For the player, this is the problem of taking the most advantageous pure strategy. Abstractly, it is PSNES standing against PSPES: equilibrium versus unstable profitability [1, 2]. Theoretically, PSPES is thought to be more profitable than PSNES, but it depends on the class of game and range of payoffs.

Goal

For generalizing, the range of payoffs in game (1) is stated via constraint

$$\mathbf{P}_i \in \mathcal{P} \subset \mathbb{R}^{\mathscr{F}} \quad \forall i = \overline{1, n} \ \text{ by } \ \mathcal{P} \bigcap \mathbb{R}^{\mathscr{F}} \neq \mathbb{R}^{\mathscr{F}}, \tag{3}$$

where $\mathbb{R}^{\mathscr{F}}$ is the space of n-dimensional $\mathscr{F}$-matrices of real elements. Subspace $\mathcal{P} \subset \mathbb{R}^{\mathscr{F}}$ is stochastic, depending on a technical event interaction which is modeled with the game (1). Stochasticity is either normal or uniform. This models an aggregate of payoffs, containing fines (negative payoff) and rewards (stimulation with positive payoff). For uniformity with nonpositive payoffs, zero payoff refers to absence of losses (stimulation whose value is shifted to the left). The goal is to ascertain the statistical relationship between averaged mean (AM) and maximal mean (MM) payoffs in PSNES and PSPES for each type of $\mathcal{P} \subset \mathbb{R}^{\mathscr{F}}$.

Experiment

While payoffs stochasticity is normal,

$$\mathcal{P} = \left\{\mathbf{P} = [p_J]_{\mathscr{F}} \in \mathbb{R}^{\mathscr{F}} : \left(\alpha^{-1} \cdot p_J\right) \in \mathcal{N}(0, 1),\ \alpha > 0\right\} \subset \mathbb{R}^{\mathscr{F}} \tag{4}$$

by the set $\mathcal{N}(0, 1)$ of values of standard normal variate. For payoffs rounded towards integers,

$$\mathcal{P} = \left\{\mathbf{P} = [p_J]_{\mathscr{F}} \in \mathbb{R}^{\mathscr{F}} : p_J \in \mathbb{Z},\ \tilde{p}_J - 1 \leqslant p_J \leqslant \tilde{p}_J,\ \left(\alpha^{-1} \cdot \tilde{p}_J\right) \in \mathcal{N}(0, 1),\ \alpha > 0\right\} \subset \mathbb{R}^{\mathscr{F}}. \tag{5}$$

If $\mathcal{U}([0; 1])$ is the set of values of uniformly distributed variate on unit segment $[0; 1]$ then

$$\mathcal{P} = \left\{\mathbf{P} = [p_J]_{\mathscr{F}} \in \mathbb{R}^{\mathscr{F}} : \left(-1 \cdot \alpha^{-1} \cdot p_J\right) \in \mathcal{U}([0; 1]),\ \alpha > 0\right\} \subset \mathbb{R}^{\mathscr{F}} \tag{6}$$

or, for rounded payoffs,

$$\mathcal{P} = \left\{\mathbf{P} = [p_J]_{\mathscr{F}} \in \mathbb{R}^{\mathscr{F}} : p_J \in \mathbb{Z},\ \tilde{p}_J - 1 \leqslant p_J \leqslant \tilde{p}_J,\ \left(-1 \cdot \alpha^{-1} \cdot \tilde{p}_J\right) \in \mathcal{U}([0; 1]),\ \alpha > 0\right\} \subset \mathbb{R}^{\mathscr{F}}. \tag{7}$$

Let $p_{\text{Nash}}(n, i, m_1, r_1)$ be payoff of the i-th player in m_1-th PSNES in the game (1), having at least a PSNES after the r_1-th generation under one of constraints (4) — (7), where $m_1 = \overline{1, M_1(r_1)}$ and $r_1 = \overline{1, R_1}$ by $M_1(r_1) \in \mathbb{N}$. And let $p_{\text{Pareto}}(n, i, m_2, r_2)$ be payoff of the i-th player in m_2-th PSPES in the game (1), having at least a PSPES after the r_2-th generation under one of constraints (4) — (7), where $m_2 = \overline{1, M_2(r_2)}$ and $r_2 = \overline{1, R_2}$ by $M_2(r_2) \in \mathbb{N}$. Integers R_1 and R_2 shall be taken sufficiently great for making statistical decision.

Averaging over game generations, AM and MM payoffs in PSNES and PSPES are

$$\begin{gathered} \bar{p}_{\text{Nash}}(n) = \frac{1}{R_1 M_1 n} \sum_{r_1=1}^{R_1} \sum_{m_1=1}^{M_1} \sum_{i=1}^{n} p_{\text{Nash}}(n, i, m_1, r_1), \quad \hat{p}_{\text{Nash}}(n) = \frac{1}{R_1} \sum_{r_1=1}^{R_1} \left(\max_{m_1 = \overline{1, M_1}} \frac{1}{n} \sum_{i=1}^{n} p_{\text{Nash}}(n, i, m_1, r_1) \right), \\ \bar{p}_{\text{Pareto}}(n) = \frac{1}{R_2 M_2 n} \sum_{r_2=1}^{R_2} \sum_{m_2=1}^{M_2} \sum_{i=1}^{n} p_{\text{Pareto}}(n, i, m_2, r_2), \quad \hat{p}_{\text{Pareto}}(n) = \frac{1}{R_2} \sum_{r_2=1}^{R_2} \left(\max_{m_2 = \overline{1, M_2}} \frac{1}{n} \sum_{i=1}^{n} p_{\text{Pareto}}(n, i, m_2, r_2) \right). \end{gathered} \tag{8}$$

Estimations (8) for subspaces (4) — (7) in constraint (3) are shown in Figures 1 — 4 along with occurrences of number of total generations R_1 within all prime 2000 generations (some of which have had not any PSNES). These barred graphs allow to affirm the following:

1. Every generated game (1) from those 64000 ones has had a PSPES.
2. AM and MM payoffs in PSNES are not tied to number of players in the game (1). Difference between normal and uniform constraints is imperceptible.
3. AM payoffs in PSPES decrease as number of players in the game (1) increases. The decreasing is faster

when constraining payoffs normally.

4. MM payoffs in PSPES increase as number of players in the game (1) increases. The increasing is slow having, seemingly, saturation effect.

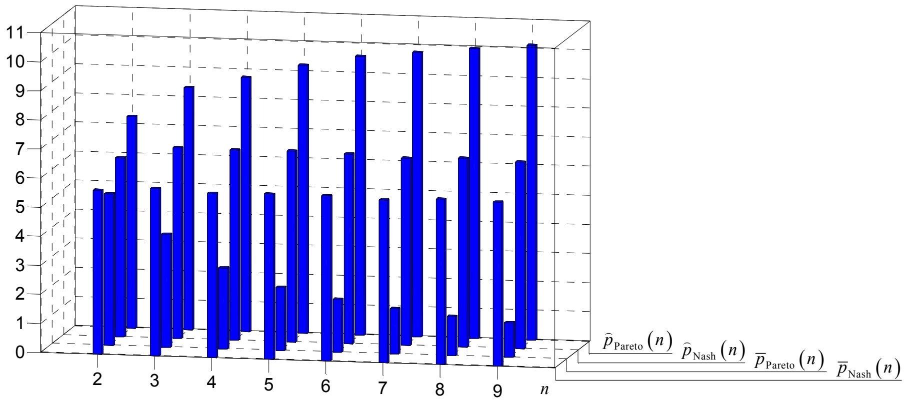

Fig. 1. Estimations (8) of AM and MM payoffs in PSNES and PSPES against number of players for (4) by $\alpha = 10$, $R_2 = 2000$

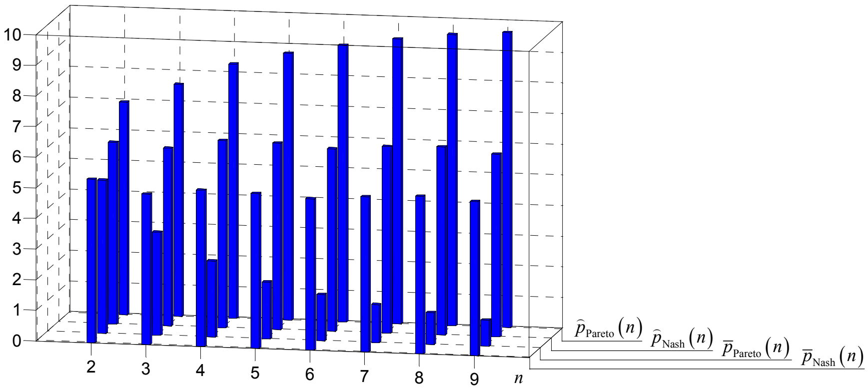

Fig. 2. Estimations (8) of AM and MM payoffs in PSNES and PSPES against number of players for (5) by $\alpha = 10$, $R_2 = 2000$

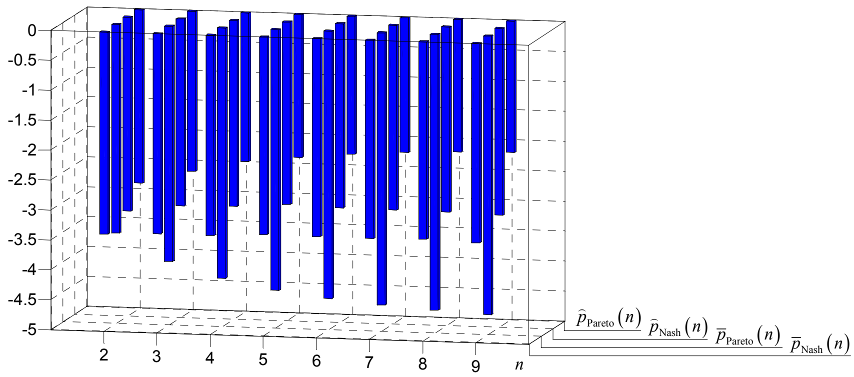

Fig. 3. Estimations (8) of AM and MM payoffs in PSNES and PSPES against number of players for (6) by $\alpha = 10$, $R_2 = 2000$

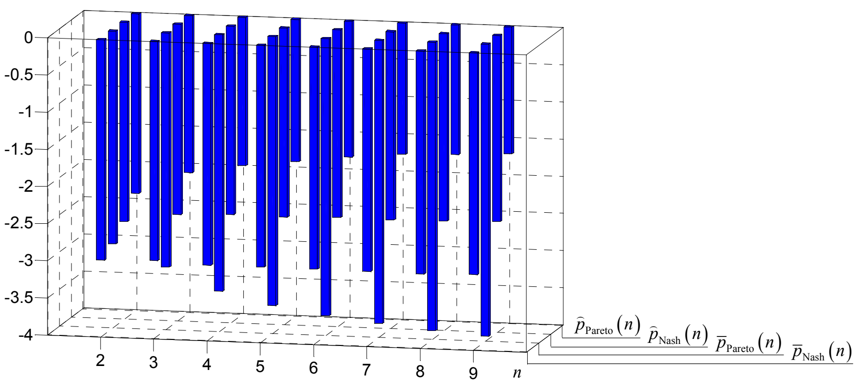

Fig. 4. Estimations (8) of AM and MM payoffs in PSNES and PSPES against number of players for (7) by $\alpha = 10$, $R_2 = 2000$

From the point of view of profitability, constraining payoffs normally is preferable to constraining them uniformly. Meanwhile, constraints (4) and (5) have roughly similar efficiency. And the most profitable case is at $n = 9$ for MM payoffs in PSPES.

When averaging totally, the advantageousness of MM payoffs in PSPES is obvious again:

$$\overline{\widehat{p}}_{\text{Pareto}} = \frac{1}{8}\sum_{n=2}^{9} \widehat{p}_{\text{Pareto}}(n) > \overline{\widehat{p}}_{\text{Nash}} = \frac{1}{8}\sum_{n=2}^{9} \widehat{p}_{\text{Nash}}(n) > \overline{\overline{p}}_{\text{Nash}} = \frac{1}{8}\sum_{n=2}^{9} \overline{p}_{\text{Nash}}(n) > \overline{\overline{p}}_{\text{Pareto}} = \frac{1}{8}\sum_{n=2}^{9} \overline{p}_{\text{Pareto}}(n). \quad (9)$$

The inequalities (9) are interpreted visually in Figure 5 for 10 cycles of 200 prime generations. Clearly, whatever the constraint law is, MM payoffs are greater than AM payoffs on average.

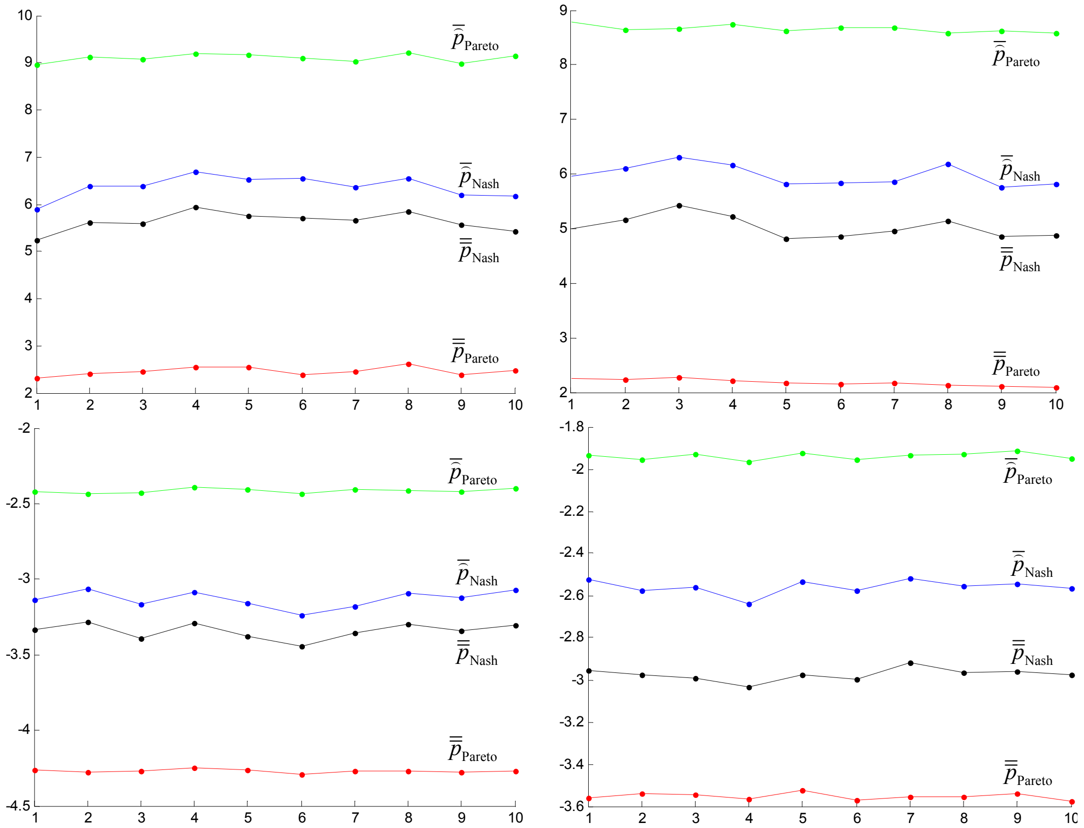

Fig. 5. Consecutive estimations of the inequalities (9) for 10 cycles (scored at abscissas) of 200 prime generations by $\alpha = 10$ for subspaces (4) — (7) in constraint (3)

Discussion and conclusion

The seeming effect of saturation of MM payoffs in PSPES hints at that the effective collective work can require up to 10 participants (workers). Not all of them, clearly, are enabled (via their "turn-on" pure strategies) at a moment. However, they may arrange who is enabled for a while. Eventually, the aggregate profitability is divided on workers if all have kept the arrangement. This decreases likelihood of a service denial in unsupervised frequency allocation, in manufacturing resources consumption, in group machinery control, etc. Besides, it is expected that with increasing number of workers the maximal Pareto efficiency will grow. Nonetheless, the greater number of workers, the harder arrangement is to be kept. The advantageousness of MM payoffs in PSPES can be applied for projecting collective work and resources usage at more efficient rate. The necessary condition is the arrangement or treaty. Although, PSPES are not necessarily equilibrium or symmetrical, the priority of PSPES is ensured with that its violation is disadvantageous as the violation can be penalized as well.

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