

## АВТОМАТИЗАЦІЯ, ТЕЛЕКОМУНІКАЦІЇ ТА РАДІОТЕХНІКА

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# THE NEW BASIC REALIZATIONS OF OPERATIONS “EQUIVALENCE” OF NEURO-FUZZY AND BIOINSPIRED NEURO-LOGICS TO CREATE HARDWARE ACCELERATORS OF ADVANCED EQUIVALENTAL MODELS OF NEURAL STRUCTURES AND MACHINE VISION SYSTEMS

*The perspective of neural networks equivalental models (EM) base on vector-matrix procedure with basic operations of continuous and neuro-fuzzy logic (equivalence, absolute difference) are shown. Capacity on base EMs exceeded the amount of neurons in 4-10 times. This is larger than others neural networks paradigms. Amount neurons of this neural networks on base EMs may be 10 – 100 thousand. The base operations in EMs are normalized equivalence operations. The family of new operations “equivalence” and “non-equivalence” of neuro-fuzzy logic’s, which we have elaborated on the based of such generalized operations of fuzzy-logic’s as fuzzy negation, t-norm and s-norm are shown. Generalized rules of construction of new functions (operations) “equivalence” which uses operations of t-norm and s-norm to fuzzy negation are proposed. Despite the wide variety of types of operations on fuzzy sets and fuzzy relations and the related variety of new synthesized equivalence operations based on them, it is possible and necessary to select basic operations, taking into account their functional completeness in the corresponding algebras of continuous logic, as well as their most effective circuitry implementations. Among these elements the following should be underlined: 1) the element which fulfills the operation of limited difference; 2) the element which algebraic product (intensifier with controlled coefficient of transmission or multiplier of analog signals); 3) the element which fulfills a sample summarizing (uniting) of signals (including the one during normalizing). The basic element of pixel cells for the construction of hardware accelerators EM NM is a node on the current-reflecting mirrors (CM), which implements the operation of a limited difference (LD) of continuous logic (CL). Synthesized structures which realize on the basic of these elements the whole spectrum of required operations: t-norm, s-norm and new operations – “equivalence” are shown. These realizations on the basic of CMOS transistors current mirror represent the circuit with analog and time-pulse optical input signals. Possibilities of “equivalence” circuits synthesis by such functions limited difference cells are shown. Such circuits consist of several dozen CMOS transistors, have low power supply voltage (1.8...3.3V), the range of an input photocurrent is 0.1...24  $\mu$ A, the transformation time is less than 1  $\mu$ s, low power consumption (microwatts). The circuits and the simulation results of their design with OrCAD are shown.*

*Keywords: self-learning equivalent-convolutional neural structures, equivalent models, continuous-logical operations, hardware accelerator, bioinspired neuro-logic, neuro-fuzzy logic, neuron-equivalentor, current mirror, sorting node, operations “equivalence” and “non-equivalence”, functional completeness, image processor.*

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## НОВІ БАЗИСНІ РЕАЛІЗАЦІЇ ОПЕРАЦІЙ «ЕКВІВАЛЕНТНІСТЬ» НЕЙРО-НЕЧІТКОЇ ТА БІОІНСПІРОВАНОЇ НЕЙРО-ЛОГІКИ ДЛЯ СТВОРЕННЯ АПАРАТУРНИХ ПРИСКОРЮВАЧІВ ПРОГРЕСИВНИХ ЕКВІВАЛЕНТНИХ МОДЕЛЕЙ НЕЙРОННИХ СТРУКТУР ТА МАШИННОГО ЗОРУ

*Показано перспективу еквівалентних моделей (ЕМ) нейронних мереж на основі векторно-матричних процедур з операціями неперервної та нейро-нечіткої логіки (еквівалентність, абсолютна різниця). Ємність нейромереж на базі ЕМ перевищувала кількість нейронів у 4-10 разів. Це більше, ніж інші парадигми нейронних мереж (НМ). Кількість нейронів таких НМ на базі ЕМ може становити 10-100 тисяч, а їх базові операції - це нормалізовані операції еквівалентності. Показано сімейство нових операцій «еквівалентність» та «нееквівалентність» нейро-нечіткої логіки, що розроблені на основі узагальнених операцій нечіткої логіки, таких як нечітке доповнення, t-норма та s-норма. Запропоновані правила побудови нових узагальнених функцій (операцій) «еквівалентності» на основі використання операцій t-норм, s-норм та нечіткого доповнення. Незважаючи на велику різноманітність типів операцій над нечіткими множинами та нечіткими відношеннями та пов'язану з ними різноманітність нових синтезованих на їх основі операцій еквівалентності, в роботі обґрунтована можливість та необхідність для їх синтезу вибирати лише три основні базові операції з урахуванням їх функціональної повноти у відповідних алгебрах неперервної логіки та елементи для їх найбільш ефективних схемних реалізацій. До цих елементів віднесено: 1) елемент, що виконує операцію обмеженої різниці; 2) елемент алгебраїчного добутку (підсилювач з регульованим коефіцієнтом передачі або помножувач аналогових сигналів); 3) елемент, що виконує зважене підсумовування (об'єднання) сигналів (у тому числі при їх нормалізації).*

Базовим елементом піксельних комірок для побудови апаратних прискорювачів ЕМ НМ є вузол на від-дзеркалювачах струму (ВДС), що реалізує операцію обмеженої різниці неперервної логіки. На основі цих базових елементів спроектовані та наведені багатofункціональні структури, які реалізують практично увесь спектр необхідних операцій: *t*-норм, *s*-норм та усі нові види узагальнених операцій «еквівалентності». Вони реалізовані на основі КМОП-транзисторних віддзеркалювачів струму та являють собою схеми нелінійної обробки аналогових чи часо-імпульсно-кодованих вхідних сигналів. Показано можливості синтезу схем «еквівалентності» за допомогою сукупності з подібних елементів, що реалізують лише операцію обмеженої різниці. Такі схеми складаються з кількох десятків КМОП-транзисторів, мають низьку напругу живлення (1,8...3,3 В), діапазон вхідного фотоструму 0,1...24 мкА, час трансформації менше 1 мкс, низьке споживання електроенергії (мікроват). Наведені спроектовані структури, схеми їх базових вузлів. Показані результати їх моделювання за допомогою OrCAD. Такі структури мають ряд переваг: однорідність, високу швидкість та надійність, простоту схем, мале енергоспоживання, високий рівень інтеграції для лінійних та матричних структур. На основі набору таких вузлів та ВДС, реалізованих за КМОП-технологією, запропоновані та розглянуті методи побудови базових елементів-комірок скалярно-реляційних векторних процесорів (СРВП) з функціями «еквівалентності» та їх структур в цілому, як прискорювачів.

Ключові слова: еквівалентно-згорткові нейронні структури, що самонавчаються, еквівалентні моделі, безперервно-логічні операції, апаратний прискорювач, біоінспірована нейрологіка, нейронечітка логіка, нейрон-еквівалентор, віддзеркалювач струму, вузол сортування, операції «еквівалентність» та «нееквівалентність», процесор зображення, нелінійна обробка, функціональна повнота.

## Introduction

For creation of advanced smart biometric systems, machine vision systems are necessary to solve the problem of object recognition in images. For the solution of recognition problems, as well as for the formation of templates spaces and classes alphabets, while development and application of recognition algorithms by a templates set of *s*, including parametric, nonparametric, neurocomputer and linguistic algorithms, while collection of patterns statistics by of experiments and mathematical or physical modeling means, that is regarded as recognition teaching and adaptation to specific conditions etc., a number of mathematics theories and methods is involved. The development of these theories and methods is connected with the advent of expert and intelligence systems: theory of fuzzy sets and theory of possibilities [1], theory of neuro networks [2, 3], and other mathematics methods [4]. More purposefully the apparatus of algebraic and fuzzy sets of continuous logic (CL); its derivatives and generalizations, including hybrid, fuzzy predicate logic, bioinspired neuro logic etc. [5–8] have become to be applied. Among non-parallel algorithms of multi-objects recognition, an important role is played by numerous versions of algorithms, based on distances calculations (Euclidian, Mahalanobise, Hamming etc) [9]. But widely spread method, used in static recognition, cluster analysis and other spheres, namely the method of classification by minimal distances, has a drawback, due to non-sufficient fast calculation of necessary distance between images, because these images are represented by vectors or matrixs with large dimension. Besides, quality of recognition, in particular, the number of stored and correctly recognized patterns in neuro-associative memory, depends greatly on chosen space and minimal distances type. In neuro-networks models and recognition algorithms in hidden layers, minimal distances (while teaching) [2, 10] and criteria of maximum similarities (equivalence) are used as intermediate criteria, in some new equivalence models (EM) for recognition of strongly correlated images [11, 12]. The interest to these new research directions, neuro-fuzzy models, logic-algebraic apparatus, common neuro-bionic principles can be explained by the possibility to understand with their help the principles of human brain functioning. As neurophysiological studies show, frequency-dynamic model of neurons (FDMN), would be the most adequate (nowadays), and neuro-biologic (NBL) as the most generalized (as compared with other known logics, such as hybrid, continuous, threshold etc.) could be used to describe the functioning of FDMN [6, 7, 13–15]. Realizations of such FDMN are already known [14–16], but they are rarely used, that is connected with their electronic realizations. Only new optic and optoelectronics technologies and their corresponding realization of neuro-modules (arrays) with a great number of circuitry models of neurons and simplified parallel one-step inputs-outputs, efficient realizations of interconnections by means of holograms or other optic methods is the most promising direction, permitting to decrease considerably the size and weight parameters, at the same time increasing the dimensions of arrays, performance and speed of data processing [7, 12, 17–19]. Basic operation of NBL, in authors opinion [11, 18, 20] or the set of operations may be not one, even functionally complete operation but also optimal, basic set of operations, which is necessary for most efficient algorithmic (and sequential in time and parallel simultaneously in space) execution of needed transformations over information signals. We realize that on this way it is quite impossible immediately to find out really final decision for all cases of infinite, complex and dynamic existence of nervous system of living beings. But consideration of such neurobiological models and their basic fundamental operations even for scalar NBL is of gnosological aspect, it permits still at the stages of conceptual approaches and structural-functional and mathematical-model design rely on vitality and a ability to compete of the suggested solutions.

Discriminant measure of the mutual alignment reference fragment with the current image, the coordinate offset is often a mutual 2D correlation function. In papers [11, 21] it was shown that to improve accuracy and probability indicators with strong correlation obstacle-damaged image, it is desirable to use methods of combining images based on mutual equivalently 2D spatial functions and equivalence models (EMs), nonlinear transformations of adaptive-correlation weighting. For the recognition, clustering of images, various models of neural networks (NN), auto-associative memory (AAM) and hetero-associative memory (HAM) are also used [22, 23]. The EM has such advantages as a significant increase in the memory capacity and the possibility of maintaining strongly correlated patterns of considerable dimensionality. Mathematical models and implement of HAM based on EMs and their modification described in papers [23, 24]. For of analysis and recognition should be solved the problem of clustering of different objects [24]. Hardware implementations of these models are based on structures, including

matrix-tensor multipliers, equivalentors [25]. And the latter are basic operations in the most promising paradigms of convolutional neural networks (CNN) with deep learning [26-28, 29]. Jim Cruchfield of UC Davis and his team are exploring a new approach to machine learning based on pattern discovery. Scientists create algorithms to identify previously unknown structures in data, including those whose complexity exceeds human understanding. In paper [30] we showed that the self-learning concept works with directly multi-level images without processing the bitmaps. But, as will be explained below, for all progressive models and concepts, nonlinear transformations of signals, image pixel intensities are necessary.

A brief overview of the mathematical operators that are implemented by neurons leads to the following conclusion. Almost all models of NN, CNN [17, 26, 27], with rare exceptions [23, 24, 29, 30], use mathematical models of neurons, which are reduced to the presence of two basic mathematical components-operators: the first component computes a function from two vectors and the second component corresponds to nonlinear transformation of the output value of the first component to the output signal. The input operator can be implemented as sum, maximal or minimum value, product of the self-weighted inputs. A lot of works has been devoted to the design of hardware devices that realize the functions of activation of neurons, but they do not consider the design of exactly the auto-equivalent transformation functions for EMs and the most common arbitrary types and types of nonlinear transformations, taking into account the limitations, we do not provide links here. In work [30], the question of the simplest approximations of auto-equivalence functions (three-piece approximation with a floating threshold) was partially solved. The basic cell of this approximation consisted of only 18–20 transistors and allowed to work with a conversion time of 1 to 2.5  $\mu$ s. At the same time, general theoretical approaches to the construction of any nonlinear type of intensity transformation were recently considered in our article [31]. The strategic direction of solution of various scientific problems image recognition becomes fast-acting and parallel processing of large data arrays (2-D) using non-conventional computational MIMO-systems, corresponding matrix logics and corresponding mathematical apparatus [32–35]. For numerous perspective realizations of optical learning neural networks (NN) with two dimensional structure [32–33, 35], of recurrent optical NN, of the continuous logic equivalency models (CLEM) NN [29, 30, 36], the elements of matrix logic are required, and not only of two-valued, but also continuous, neural-fuzzy logics. Advanced parallel computing structures and MIMO-systems require both parallel processing and parallel input/output of information [37, 38, 39]. Generalization of scalar two-valued logic on matrix case has led to intensive development of binary images algebra (BIA) and 2D Boolean elements for optic and optoelectronic processors [17, 23–25, 32–33, 35, 36, 40]. Taking into consideration the above-described approach, consisting in universality, let us recollect some known facts regarding the number of functions [37]. Thus, the search of means aimed at construction of elements, especially universal or multifunctional with programmable tuning, able to perform matrix (multi-valued, continuous) logic operations is very actual problem. One of promising directions of research in this sphere is the application of time-pulse-coded architectures (TPCA), that were considered in work [37], which contains links to our previous work regarding this idea. In addition, the analysis of the functional completeness of various algebras of logics allowed us to offer a useful idea in which it is possible to create more functional processors that use a specific coding of time pulse operands and only two-valued logic elements to implement all the functions of different analog logics. But, in the implementations of this approach, there is an acute problem of synchronization of time pulses and increased computation times. Therefore, for a more rigorous theoretical foundation and our other approach, which consists in using continuous logic circuits, but with analog amplitude and without time pulse coding, based on current mirrors with photodiodes and light-emitters for input-output of variables, we need to define such a basic set of operations, which would significantly expand the range of functions implemented with the help of such basic elements for all possible varieties of analog and hybrid logics. And this is the purpose of our work. We will note that on a current mirror more easily to execute these operations of addition or subtraction of currents.

**1. Research problem setting.** Creation of EMs NAM and ANN including: high order for recognition highly correlated images with adaptive double (adaptive-equivalence, adaptive-correlative) weighting of new modified matrix-tensor neurologic equivalence models (MTNLEMs) with double adaptive-equivalence weighting (DAEW) for space-non-invariant recognition (SNIR) and space-invariant recognition (SIR) of 2-D patterns permitted to discuss the problem of creation of new equivalence paradigm ANN of non-iterative type [33, 41]. Computation process in such models is reduced to by-step procedures (vector-matrix, matrix-tensor) of determination in most general form (for SIR) space-dependent normalized equivalence (non- equivalence) functions from two images or simply (for SNIR) normalized equivalences (non equivalences) of two matrix. In [33, 41–43] the advantages and efficiency of such EMs models are demonstrated. The result of experimental research [23, 24, 33, 43] confirmed the advantages of these models and, in particular, the increase of ANN volume in case of such MTNLEMs, at least to  $(1.1-4.0)N$ , where  $N$  – number of neurons ANN! But at the same time the choice of equivalence (non- equivalence) operations, their types were heuristic and there was no close connection of EA and NBL with fuzzy-logic. That is why the aim of the given research is the consideration of new equivalence (non-equivalence) operations as well as their possible optoelectronics realizations. The suggested family of such new operations will permit in each given case-to select more efficient variant, equivalence type and will enlarge the range of research in the paradigm of “equivalence”. For all known convolutional neural networks, as for our EMs, it is necessary to calculate the convolution of the current fragment of the image in each layer with a large number of templates that are used, which are a set of standards that are selected or formed during the learning process. But, as studies show, large images require a large number of filters to process images, and the size of the filters can also be large. There is an acute problem of increasing the computational performance of hardware implementations of such

CNNs. Therefore, the last decade has been marked by the intensification of work on the creation of specialized neural accelerators, and we also want to take part in this process, make our modest contribution and propose a new conceptual structure of the accelerator just for our equivalence paradigm [23-25, 33, 36, 40, 49].

**2. Fuzzy set theory and fundamental operations of fuzzy logic.** Common theory of sets is based on classical two-valued logic with its law of elimination of third value, that is why characteristic function  $\mu_c(x)$  of  ${}^2C$  set takes only two values: zero if  $x \notin {}^2C$  units (if  $x \in {}^2C$ ). But greater part of classes of objects, existing in the real world can't be determined accurately. For such classes characteristic function of belonging is equal zero only for several elements (element doesn't belong to the class) and must be intermediate between zero and unit of belonging value. Such classes of objects are the object of fuzzy set theory investigation. In common case in order to determine the fuzzy set the refraction of universal set of objects  ${}^{\wedge}C$  in the section  $[0, 1]$  is used:  $\mu_A(x) : {}^{\wedge}C \rightarrow [0, 1]$ , determining for each  $x \in {}^{\wedge}C$  its degree of belonging to fuzzy set A. Thus, the fuzzy set can be determined as the set of pairs  $(x, \mu_A(x))$ , where  $x \in {}^{\wedge}C$ ,  $\mu_A(x)$  – belonging function. In other words, fuzzy subsets A of the set  ${}^{\wedge}C$  are called the set of arranged pair of the form  $(x, \mu_A(x))$  where  $x \in {}^{\wedge}C$ ,  $\mu_A(x)$  – function determined on  ${}^{\wedge}C$ , and the carriers of fuzzy subset A (supp A) is called the set (in common sense) of elements x, for which the following is valid:

$$\text{supp } A = \{x | \mu_A(x) > 0\}, x \in {}^{\wedge}C. \quad (1)$$

Let  ${}^{\wedge}C$  be the universe of discourse, with a generic element of  ${}^{\wedge}C$  denoted by x. A fuzzy subset A is a collection of elements in  ${}^{\wedge}C$  such that with element x is associated a degree of membership  $\mu_A(x)$ . Mathematically, the definition of the fuzzy subset A can be written as  $A = \{x, \mu_A(x)\}$ . The value  $\mu_A(x) = 0$  represents no membership, and  $\mu_A(x) = 1$  represents full membership. If only no membership and full membership are allowed, then A is a crisp set. Thus fuzzy subsets can be considered as an extension of traditional crisp set, of in other words, the classical crisp set is a special case of a fuzzy set [8].

Assuming there are two fuzzy subsets  $A = \{x, \mu_A(x)\}$  and  $B = \{x, \mu_B(x)\}$ ,  $x \in {}^{\wedge}C$  some common operations are defined as follows [44, 45]:

1. Complement (fuzzy equivalent to crisp-logic NOT):  $\bar{A}$ , with  $\mu_{\bar{A}}(x) = 1 - \mu_A(x)$ ,  $\forall x \in {}^{\wedge}C$  (2)

2. Union of fuzzy subsets A and B (fuzzy equivalent to crisp-logic OR):  $A \cup B$  are determined in the following way:  $\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x)) \in {}^{\wedge}C = [0, 1]$ ,  $\forall x \in {}^{\wedge}C$  (3)

3. Intersection (fuzzy equivalent to crisp-logic AND):  $A \cap B$   
 $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$ ,  $\forall x \in {}^{\wedge}C$  (4)

4. Equalization:  $A = B$   $\mu_A(x) = \mu_B(x)$ ,  $\forall x \in {}^{\wedge}C$  (5)

5. Containment:  $A \subset B$  if  $\mu_A(x) \leq \mu_B(x)$ ,  $\forall x \in {}^{\wedge}C$  (6)

6. Symmetric difference (fuzzy equivalent to crisp-logic OR, XOR):  $A \Delta B$  with  
 $\mu_{A \Delta B}(x) = |\mu_A(x) - \mu_B(x)|$   $\forall x \in {}^{\wedge}C$  (7)

First, suggested by L. Zadeh [1] system of operations over fuzzy sets comprised operations (1÷7). The theory of fuzzy sets provides greater possibilities while constructing and selection of functions for representation of theoretic – set  $(\cap, \cup, -)$  and correspondingly logic (AND, OR, NOT) operations. Fuzzy measure is the generalization of probability. Condition of additivity of probability measure for fuzzy measure is replaced by weaker condition of monotony by inclusion [46]. Generalization of basic in mathematics notion of set and introduction by L. Zadeh [47] of the principle of generalization (adaptation of standard mathematic means and grouping of fuzzy sets) promoted the success of the theory of fuzzy sets, which also has genuine connection with multivalued and modal logic. Multivalued logics, in limitation and continuous [5–7, 15, 46]. can be interpreted as fuzzy (i.e. transformed into fuzzy) standard notation of statements. That is why logic connections can be presented as functions (truth values), then they can be fuzzy applying the principle of generalization.

Later, the class of fuzzy measures was constructed, it is based on so called *t*-norms, *s*-norms [46]. Generalization of operations NOT, AND, OR of two-valued logic, in fuzzy logic are called fuzzy negation  $(-)$  *t*-norms and *s*-norms correspondingly [8, 46]. They are determined in the following way:

1. Fuzzy negation  $n$ ,  $n : [0, 1] \rightarrow [0, 1]$ , so:  $0_n = 1$ ;  $(A_n)_n = A$ ;  $A < B \Rightarrow A_n > B_n$  (8)

For instance, compliment:  $A_n = 1 - A$  (9)

2. *t*-norms  $t : [0, 1] \times [0, 1] \rightarrow [0, 1]$ , so:  $At1 = A$ ;  $At0 = 0$ ;  $A \leq B \Rightarrow AtC \leq BtC$ ;  
 $AtB = BtA$ ;  $At(BtC) = (AtC)tC$ .

The examples of  $t$ -norms are:

$$\text{– logic product (see 3 (intersection))}: A \wedge B = \min(A, B); \quad (10)$$

$$\text{– algebraic product}: A \cdot B = AB; \quad (11)$$

$$\text{– limited product}: A \dot{\times} B = 0 \vee (A + B - 1); \quad (12)$$

$$\text{– contrast product}: A \dot{\wedge} B = \begin{cases} A, & \text{if } B = 1; \\ B, & \text{if } A = 1; \\ 0, & \text{if } A, B \neq 1; \end{cases} \quad (13)$$

3.  $s$ -norms ( $t$ -conorms)  $s : [0, 1] \times [0, 1] \rightarrow [0, 1]$ , so:  $As1 = 1$ ;  $As0 = A$ ;

$$A \leq B \Rightarrow AsC \leq BsC; AsB = BsA; As(BsC) = (AsC)sC.$$

The examples of  $s$ -norms are:

$$\text{– logic sum}: A \vee B = \max(A, B); \quad (14)$$

$$\text{– algebraic sum}: A \dot{+} B = A + B - AB = A \cdot \bar{B} + B; \quad (15)$$

$$\text{– limited sum}: A \dot{\cup} B = 1 \wedge (A + B); \quad (16)$$

$$\text{– contrast sum}: A \dot{\vee} B = \begin{cases} A, & \text{if } B = 0; \\ B, & \text{if } A = 0; \\ 1, & \text{if } A, B > 0; \end{cases} \quad (17)$$

$$\text{4. Limited difference}: A \dot{-} B = 0 \vee (A - B). \quad (18)$$

Duality relations  $t$ -norms and  $s$ -norms relatively fuzzy negation represent generalized form of De Morgan law, written in the following way:  $((A_n)t(B_n))_n = AsB$  or  $((A_n)s(B_n))_n = AtB$ .

The analysis of main operations performed over fuzzy sets in terms of belonging function (see formulas 2–7) and in terms of variables of fuzzy (analogue, probable and other) logic (see formulas 10÷18), shows that all these operations are, by their nature, operations of continuous logic, or in general scalar NBL. It is of no importance for us to know what the variable is in NBL, whether it is the object itself or its belonging function or control signal etc. Thus, further it is quite sufficient to consider NBL with its basic operations or corresponding algebra's.

**3. Initial equivalence (non-equivalence) operations of NBL.** For the first time in the work [19], for constructing of models of neuronal associative memory (NAM) base binary operations of NBL “equivalence” ( $\sim$ ) and “non-equivalence” ( $\nabla$ ) were used of such types:

$$a \sim b = \min\{\max(a, \bar{b}), \max(\bar{a}, b)\} = (a \vee \bar{b}) \wedge (\bar{a} \vee b); \quad (19)$$

$$a \nabla b = \max\{\min(a, \bar{b}), \min(\bar{a}, b)\} = (a \wedge \bar{b}) \vee (\bar{a} \wedge b); \quad (20)$$

$$a \overset{+}{\sim} b = 1 - |a - b|; \quad (21)$$

$$a \overset{+}{\nabla} b = |a - b|, \text{ where } a, b \in C_u = [0, 1], a_n = 1 - a, b_n = 1 - b. \quad (22)$$

For convenience, fuzzy negation in NBL are also marked with an upper line. In works [11, 20] new equivalental operations of NBL were introduced:

$$a \overset{\sim}{\sim} b = a \cdot b + \bar{a} \cdot \bar{b}; \quad (23)$$

$$a \overset{\nabla}{\nabla} b = a \cdot \bar{b} + \bar{a} \cdot b, \quad (24)$$

with are more convenient in use, properties of there operations were determined, their connection with function of scalar with metric distances, the connection was demonstrated between “equivalental algebra” (EA) and other algebra's of continuous and multivalued logics. The terms of equivalental uni-dimension (1-D) and bi-dimension (2-D) functions were introduced:

$$E(\xi) = f(\bar{a}, \bar{b}_\xi) = \frac{1}{N} \sum_{i=1}^N (a_i \sim b_{i-\xi}), \quad (25)$$

$$\tilde{E}(\xi, \eta) = f(A, B) = A \overset{\sim}{*} B = \sum_{n=1}^N \sum_{m=1}^M (a_{n,m} \sim b_{\xi+n, \eta+m}), \quad (26)$$

terms of systemic equivalental function  $\Phi_{\sim}$  and systemic non- equivalental function  $\Phi_{\nabla} = \overline{\Phi_{\sim}}$ , with the help of which it is possible to study the basis of “equivalental” models (EMs) by analog with equalization function in known models. The connection of functions  $\tilde{E}(\xi, \eta)$  with correlation functions was demonstrated [20].

Normalized equivalence of two matrices  $A = \{a_{ij}\}_{I \times J}$  and  $B = \{b_{ij}\}_{I \times J} \in [0,1]^{I \times J}$  is determined in the following way [19, 20]:

$$A \sim_n B = \sum_{i=1}^I \sum_{j=1}^J \frac{(a_{ij} \sim b_{ij})}{I \cdot J}, \quad (27)$$

and correspondingly normalized non-equivalence:

$$A \not\sim_n B = \sum_{i=1}^I \sum_{j=1}^J \frac{(a_{ij} \not\sim b_{ij})}{I \cdot J}. \quad (28)$$

It should be noted, if  $1 = [1]_{I \times J}$  (matrix of units) then  $A \not\sim_n 1 = a_m$  (mean arithmetic value), and

$A \not\sim_n 0 = 1 - a_m = \bar{a}_m$  (complementary mean arithmetic value). Operations  $\left(\sim_n\right)$  and  $\left(\not\sim_n\right)$  are measures of similarity

(equivalence) and difference (non-equivalence, distance) of matrices, which are connected with Hamming distance, in particular [20]. Thus, by components operations  $(\sim)$  and  $(\not\sim)$  of scalar NBL are generalized on matrix case, and NBL logic becomes matrix NBL, i. e. (MNBL). The peculiarity of above considered measures or criteria on their base is that they are invariative to scale change (range) of input components of vectors, matrix, to the change of signal polarity, to the choice of coding type (unipolar of bipolar) to the change of constant component (simultaneous shift of all components by level) belong to the same range  $[-D, D]$ , are normalized and interconnected [11, 12, 20].

Without losing of community we can consider carrying set (for scalar case)  $C_u = [0,1]$  and  $C_u^N = [0,1]^N$  (for N-dimension (vector, matrix) case).

**4. Extension of family of NBL equivalence (non-equivalence) operations.** Let us introduce new generalized operation of the its type of equivalence (non-equivalence) operation, written in the following form:

$${}^{t,s}E'(a,b) = (atb)s(a_n tb_n) = (atb)s(\bar{a}t\bar{b}), \quad (29)$$

and as  $t$ -norm and  $s$ -norm any of their variants can be used (according to 10÷17). The analysis of the whole spectrum of all possible  ${}^{t,s}E'(a,b)$  shows, that the following operations are the most interesting in case of certain  $t$ - and  $s$ -norms:

$$\bullet, + E'(a,b) = a \cdot b + \bar{a} \cdot \bar{b} = a \sim b, \text{ (known operation see formula (23))} \quad (30)$$

$$\bullet, + E'(a,b) = a \cdot b + \bar{a} \cdot \bar{b} - a \cdot b \cdot \bar{a} \cdot \bar{b} = (a \sim b) - (a \cdot b) \cdot (\bar{a} \cdot \bar{b}), \text{ (new operation)} \quad (31)$$

$$\bullet, \vee E'(a,b) = a \cdot b \vee \bar{a} \cdot \bar{b} = \max(a \cdot b, \bar{a} \cdot \bar{b}), \text{ (new operation)} \quad (32)$$

$$\wedge, + E'(a,b) = \min(a,b) + \min(\bar{a}, \bar{b}) = a \sim b, \text{ (see formula (21) known operation)} \quad (33)$$

$$\wedge, \vee E'(a,b) = a \sim b = (a \wedge b) \vee (\bar{a} \wedge \bar{b}), \text{ (see formula (19) known operation)} \quad (34)$$

$$\begin{aligned} \wedge, + E'(a,b) &= (a \wedge b) \div (\bar{a} \wedge \bar{b}) = \min(a,b) + \min(\bar{a}, \bar{b}) - \min(a,b) \cdot \min(\bar{a}, \bar{b}) = \\ &= \left(a \sim b\right) - \min(a,b) \cdot \min(\bar{a}, \bar{b}), \end{aligned} \quad \text{(new)} \quad (35)$$

It should be noted, that for the case of taking the complement by one the variables, these operations will have the following form:  $\bullet, + E'(a, \bar{b}) = a \cdot \bar{b} + \bar{a} \cdot b = a \not\sim b = \bullet, + E'(\bar{a}, b)$ , (see formula (24))

$$\bullet, + E'(a, \bar{b}) = a \cdot \bar{b} + \bar{a} \cdot b - (a \cdot \bar{b}) \cdot (\bar{a} \cdot b) = (a \not\sim b) - (a \cdot b) \cdot (\bar{a} \cdot \bar{b}), \quad (37)$$

$$\bullet, \vee E'(a, \bar{b}) = a \cdot \bar{b} \vee \bar{a} \cdot b = \max(a \cdot \bar{b}, \bar{a} \cdot b), \quad (38)$$

$$\wedge, + E'(a, \bar{b}) = (a \wedge \bar{b}) + (\bar{a} \wedge b), \quad (39)$$

$$\wedge, \vee E'(a, \bar{b}) = (a \wedge \bar{b}) \vee (\bar{a} \wedge b) = a \not\sim b, \text{ (see formula (20) known operation)} \quad (40)$$

$$\wedge, + E'(a, \bar{b}) = (a \wedge \bar{b}) + (\bar{a} \wedge b) - \min(a, \bar{b}) \cdot \min(\bar{a}, b) = \wedge, + E'(a, \bar{b}) - \min(a, \bar{b}) \cdot \min(\bar{a}, b) \quad (41)$$

Introducing new generalized operation of equivalence of II type (non-equivalence) we write in the following form:  ${}^{s,t}E''(a,b) = (asb)\not\sim(\bar{a}s\bar{b})$

or taking into consideration the law of De Morgan so:

$${}^{s,t}E''(a,b) = ((asb)_n s(\bar{a}s\bar{b})_n) = ((\bar{a}t\bar{b})_n s(atb)_n) = ({}^{t,s}E'(a,b))_n = \overline{{}^{t,s}E'(a,b)} \quad (43)$$

we will obtain the connection between operations of I and II type. That is why, the II type of operations can be called the operation “non-equivalence” of the I type and designate it as:

$${}^{t,s}NE'(a,b) = {}^{s,t}E''(a,b) = ({}^{t,s}E'(a,b))_n \quad (44)$$

Thus, function (29) and function (43), which is a complement to function (29), define new generalized comparison operations (definition of equivalence or nonequivalence).

##### 5. Definition and selection of NBL basic operations for functionally complete systems.

It can be shown that formula (11) is reduced to the form:  $a \cdot b = 1 - (\bar{a} \dot{+} \bar{b}) = (\bar{a} \dot{+} \bar{b})_n = (\bar{a} \dot{+} \bar{b})$ . (45)

In the same way formula (15) is reduced to:  $(a \dot{+} b) = 1 \dot{-} (\bar{b} \cdot \bar{a})$  (46)

and formulas (16), (12), (10), (14), (7) are reduced accordingly to such forms:

$$a \dot{\cup} b = (\bar{a} \dot{\times} \bar{b})_n = 1 \dot{-} (0 \vee (\bar{a} + \bar{b} - 1)) = 1 \dot{-} (0 \vee (\bar{a} \dot{-} b)) = 1 \dot{-} (1 \dot{-} (a + b)) \quad (47)$$

$$a \dot{\times} b = (a + b) \dot{-} 1 \quad (48)$$

$$\min(a, b) = a \dot{-} (a \dot{-} b) = b \dot{-} (b \dot{-} a) \quad (49)$$

$$\max(a, b) = a + (b \dot{-} a) = b + (a \dot{-} b) \quad (50)$$

$$|a - b| = (a \dot{-} b) + (b \dot{-} a) \quad (51)$$

If we don't take into account operation  $(\wedge)$  and  $(\vee)$ , then it can be noted that all the operations (formula 2–7 and 10–17) and all new generalized operations  $({}^{t,s}E'(a, b)$  and  $({}^{t,s}NE'(a, b))$  can be realized by means of three basic operations: **simple addition**  $(+)$ , **limited difference**  $(\dot{-})$ , **product**  $(\cdot)$ . Operations  $(+)$  and  $(\cdot)$  are easily performed by optical methods, the first -  $(+)$  by means of space and time integration of light signal, and the second  $(\cdot)$  – by means of passing light signals through two serially placed analog space modulators. Analysis of the above expressions allows us to draw the following conclusion. If there is a constant "1" in continuous-logical algebras NBLs and taking into account that  $\max(a, b) = (\min(a_n, b_n))_n = 1 \dot{-} \min(a_n, b_n)$ , most types of generalized operations  $t$ -norms and  $s$ -norms can be expressed using just one operation, namely a limited (bounded) difference  $(\dot{-})$ . Therefore, it is necessary to pay special attention to improved implementations of this particular operation  $(\dot{-})$ . In addition, for new circuitry implementations of this operation, it is necessary to choose as variables or arguments such types and features of signals, methods of their representation and coding in order to meet the requirements for the accuracy of analog calculations and simultaneously fulfill a number of other characteristic requirements: speed, energy efficiency, scalability, and etc. Let us recall one more important aspect concerning operators that are implemented by neurons in neural network models. Almost all concepts, models, structures of neural networks use informational mathematical models of neurons, which are reduced to the presence of two basic mathematical components-operators: the first component computes a function from two vectors  $\bar{X}$  and  $\bar{W}$ , where  $\bar{X}$  – vector of input signals of neuron,  $\bar{W}$  – vector of weights, and the second component-operator corresponds to some nonlinear transformation of the output value of the first component to the output signal. The input operator can be implemented as many expressions [40], but the most commonly used is the sum of products: sum of the self-weighted inputs  $f(\bar{X}, \bar{W}) = \sum_{i=1}^N w_i x_i$ . Recently, the set of such operators has expanded significantly, and, for example, the emergence of equivalence models [23, 24, 33, 36] of neural networks, which have some advantages, requires the calculation of such below given operators as normalized equivalence  $({}^{t,s}e_i)$  of vectors and normalized nonequivalence  $({}^{t,s}e_i)_n$  of vectors  $\bar{X}$  and  $\bar{W}$ :

$$f(\bar{X}, \bar{W}) = \hat{e}(\bar{X}, \bar{W}) = \frac{1}{N} \sum_{i=1}^N (x_i \hat{\sim} w_i) = \frac{1}{N} \sum_{i=1}^N (x_i w_i + (1 - x_i)(1 - w_i)), \text{ or}$$

$$f(\bar{X}, \bar{W}) = \frac{1}{N} \sum_{i=1}^N (x_i \overset{\sim}{\sim} w_i) = \frac{1}{N} \sum_{i=1}^N (x_i \wedge w_i \vee \bar{x}_i \wedge \bar{w}_i) = \overset{\sim}{\sim} e(\bar{X}, \bar{W})$$

$$f(\bar{X}, \bar{W}) = \hat{ne}(\bar{X}, \bar{W}) = \frac{1}{N} \sum_{i=1}^N (x_i \hat{\not\sim} w_i) = \frac{1}{N} \sum_{i=1}^N (x_i \bar{w}_i + \bar{x}_i w_i) \text{ , or}$$

$$f(\bar{X}, \bar{W}) = \frac{1}{N} \sum_{i=1}^N (x_i \overset{\sim}{\not\sim} w_i) = \overset{\sim}{\sim} ne(\bar{X}, \bar{W}) = \frac{1}{N} \sum_{i=1}^N ((x_i \wedge \bar{w}_i) \vee (\bar{x}_i \wedge w_i))$$

Here we denote them with small letters  $e$  and  $ne = (e)_n = 1 - e$  with superscripts depending on the type of  $t$ -norms and  $s$ -norms used in the scalar element-wise equivalence and nonequivalence operations  $({}^{t,s}E'(a, b)$  and

${}^{t,s}NE'(a,b)$ ), discussed in section 4 and denoted by capital letters with upper left indices. A positive aspect of the work [40] was the use of the modular principle, which made it possible to calculate the normalized equivalence operator of two vectors by calculating the normalized partial equivalences of their subvectors, followed by a similar **calculation of the normalized equivalence** of the vector of their partial signals with a unit control vector. It shows that all algorithmic procedures in the equivalence paradigm of neural networks and hetero-associative memory on their basis are reduced to the calculation of normalized equivalences from two vectors or matrices, and the elemental nonlinear transformations of neuron output signals that correspond to the activation functions. The latter are reduced to the calculation of autoequivalences (auto-nonequivalences) for the above models of equivalence of neural networks. Consider the structural design, using the approaches in [40].

Let vectors  $\vec{X}$  and  $\vec{W}$  have dimension  $N = k \cdot Q$ . For every  $i$ -th ( $i \in 1 \div k$ ) subvector  $\vec{X}_i$  and  $i$  subvector  $\vec{W}_i$ , the dimension of which equal to  $Q$ , it is possible to calculate  $f_i(\vec{x}_i, \vec{w}_i) = \frac{1}{Q} \sum_{l=1}^Q (x_i^l \sim w_i^l) = \tilde{e}_i(\vec{x}_i, \vec{w}_i)$ . Then it is possible to show that:  $f(\vec{x}, \vec{w}) = \frac{1}{k} \sum_{i=1}^k f_i(\vec{x}_i, \vec{w}_i) = \frac{1}{k} \sum_{i=1}^k (f_i \sim 1) = \tilde{e}(\vec{f}_k, \vec{1}_k)$ , where  $\vec{f}_k$  and  $\vec{1}_k$  are vectors of  $K$  dimension, the components of which are equal  $f_i$  and "1" accordingly. Like:  $\tilde{f}(\vec{x}, \vec{w}) = \tilde{ne}(\vec{f}^k, \vec{0}^k)$ . Hence, for increase of vectors dimension, given at inputs of our complementary-dual neuron element (CDNE) with generalized equivalence operation, it is possible to use base analogical CDNEs of less dimension, creating a hierarchical structure out of them. As can be seen from the above mathematical justification, the presence of complementary outputs in basic functional nodes CDNEs, which are implemented quite simply, greatly simplifies the synthesis of larger neural-like structures at higher levels of design. Thus, to implement the basic modules for calculating normalized equivalences from two vectors or subvectors, an operation of **simple summation** of signals is required, and it must add signals from the outputs of the nodes that calculate any one selected from the set of all introduced new generalized operations ( ${}^{t,s}E'(a,b)$  and  ${}^{t,s}NE'(a,b)$ ). Additionally, it is necessary to normalize this sum by dividing it by the number of adder inputs or to weigh the adder input signals by **multiplying** them by  $1/k$  or  $1/Q$ . In addition, in order to expand the functionality, it is desirable to provide for a programmed choice of the type and type of the generalized equivalence operation, as well as the direct or complementary value of the analog argument. Taking into account the above conclusions regarding the list of required basic operations for constructing accelerators with extended functional completeness, we recall that in circuits based on CMOS current mirrors, it is easy to sum up currents and weigh them by selecting the transmission coefficients of the mirrors, all the more in different and simple ways. In addition, the input of information signals and control signals through the use of introduced photodetectors, combined with current mirrors and their current modes, increases the prospects of circuit solutions with this approach to design, "from structural to elementary physical level", and solves many technological aspects and problems interconnections and dynamic settings and programming.

## 6. Design of basic realizations of SRVP and CDNE based on generalized equivalence operation.

At the same time, after element-wise processing of vectors in such scalar-relational vector processors (SRVP) based on CDNEs, a wider set of operations  ${}^{t,s}e_i$  on the obtained vector  $\vec{E} = \{{}^{t,s}e_1, {}^{t,s}e_2, \dots, {}^{t,s}e_n\}$  of element-wise or block-wise relations is possible. Therefore, in a general view the SRVP can be described by the following model:

$$\Psi_{SRVP}(\vec{A}, \vec{B}) = f^2(\vec{E}) = f^2(e_1(a_1, b_1), \dots, e_n(a_n, b_n)) = f^2(f_1^1(a_1, b_1), \dots, f_n^1(a_n, b_n)), \quad (52)$$

where  $f^2$  - one of a set of possible functions with generalized equivalence operation, mapping a vector  $\vec{f}^1 = \{f_1^1, \dots, f_n^1\}$  into  $\Psi$ , where  $\Psi, f_i \in M = [0,1]$ , i.e.  $[0,1] \times [0,1] \times \dots \times [0,1] \rightarrow [0,1]$ , and  $f_i^1$  - one of a set of possible functions mapping a vector  $(a_i, b_i, 1 - a_i, \bar{b}_i) \rightarrow f_i$ . The SRVPs considered in this article can be applied to solve problems based on fuzzy logic and generalized NBL. For such vector processors with scalar relations, in each specific case, one can choose one or another s-norm and t-norm, but then the processor will only execute a certain procedure. This approach has little promise. Therefore, we want to allot our scalar-relation vector processors by universality (or at least quasi-universality). Therefore requirements to such processors the following: changing

type of operations at algorithmic steps ( $f^1$  at the first step,  $f^2$  at the second step), we can ensure execution of required relations.

The structural diagrams of the SRVP based on CDNE are shown in Fig.1. The CDNE are depicted as blocks, each with two vector inputs and two scalar complementary outputs  $e$  and  $ne$ . The CDNE consists of  $n$  channel-nodes of element-wise analog processing (or a group of  $n$  elements  $ULE_1-ULE_n$ ), (Fig.2), which implement  $f_i^1$  operations, and element of second stage, which implements function  $f^2$  (see formula (52)). And, the type of  $t$ -norm or  $s$ -norm operation (for function  $f_i^1$ ) can vary with the help of special adjusting signals  $\bar{C}_1 - \bar{C}_m$ , and the type of  $t$ -norm or  $s$ -norm operation (for function  $f^2$ ) can vary too with the help of special adjusting signals  $\bar{G}_1 - \bar{G}_p$ . At the input of the scalar-relation vector processor the operands like vectors  $A = \{a_i\}$  and  $B = \{b_i\}$ , ( $i = \overline{1, n}$ ), and also their complements  $A^c = \{a_i^c\}$  and  $B^c = \{b_i^c\}$  come. In structural diagram of CDNE shown in Fig.2 output signals from universal logical element of two-input (two-valued) logic (ULE TVL) of 1th –stage come to a converter (normalizer), that realizes signals integrations or simple summation (in 2-th stage). The modified conveyor, homogeneous with regular connections wave structure (MCHWS), which performs sorting and ordering of analog signals and is one of the main element-wise processing nodes, are shown in Fig. 2 (top right). In the future, for simplicity, we can call this main unit the sorting unit (SU) of analog signals, the specifics and implementation features of which depend on the type and form of signal representation. Here considered analog processors based on the SU, consisting of a conveyor of layers of selector-rank disjunctive-conjunctive elements (SRDCEs). Modified wave structure on the basis of SRDCE using the continuous logic (CL) base analog cell (BC) with many ordered outputs. In Fig. 2 (bottom right) shows an SRDCE circuit that is known and modified to better match currents in and out.

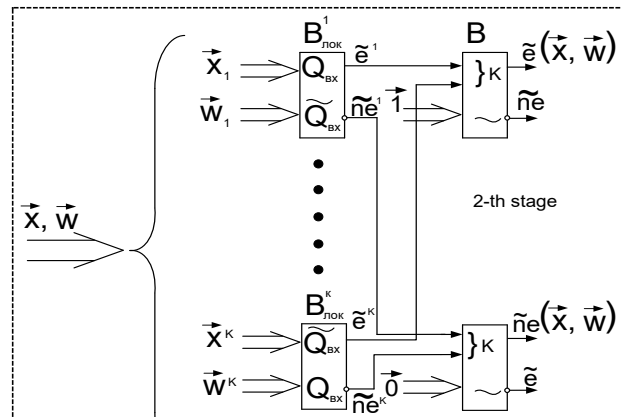


Fig. 1. The structural diagrams of the scalar-relation vector processor based on CDNE

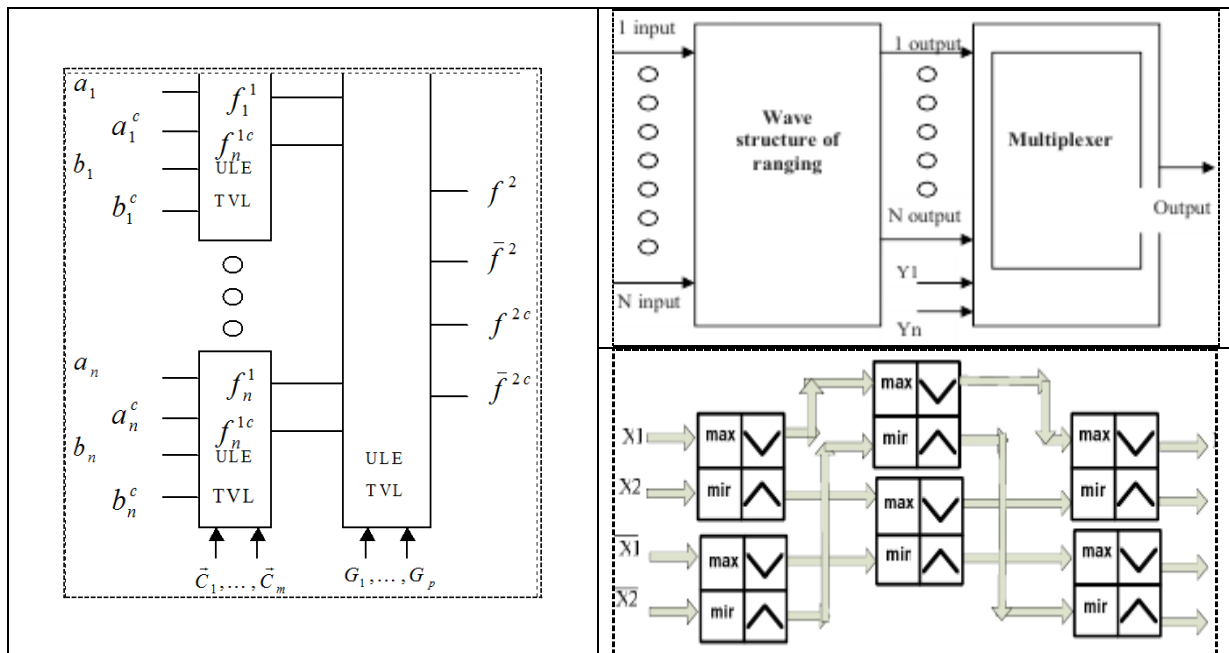


Fig. 2. The structural diagrams CDNE (left), consisting of  $n$  channel-nodes of element-wise analog processing, a conceptual structure of a node based on a rank selector in the form of a waveguide sort and multiplexer (top right) and a sorting scheme (bottom right).

The 4-input sorting unit of analog signals  $(a_i, b_i, 1 - a_i, \bar{b}_i)$  using of 6 the modified CL BCs based on two-input and two-output SRDCEs is represented here. The circuit is in fact the device of order logic, calculating simultaneously all operations of the proper ranks  $r = 1-4$ . The simulation results of such SU shown, that the structure functions correctly, although error in the levels of signals before and after do not exceed 5–6 % for currents  $D_{\max} = 20-40 \mu\text{A}$  and 1–3 % for currents  $D_{\max} = 5-10 \mu\text{A}$ . A multiplexer allows the control code ( $y_1, y_2, \dots, y_n$ ) to select rank  $n$  and, accordingly, the type of operation required. The CL BC circuits of only 13 CMOS transistors, and adding 4 or 6 transistors to them, which compare the currents, it is easy to get a digital potential output of the comparator. The SRDCE as base cell executing organization of two analog optical signals that in fact is two photocurrents was designed and simulated. The simulation results of operation of such cell and SU are shown in Fig. 3 and are following: range of the input photocurrents 0.1–40  $\mu\text{A}$ , supply voltage is 1.8–3.3 V, duration of fronts 20–100 ns, total delays no more than 200ns. Such cells are enough simple (only 13 transistors). At complication they can be more high exactness. Figure 4 shows the structural circuit of channel-nodes of element-wise analog processing, which was used for modeling in the Orcad Pspice environment.

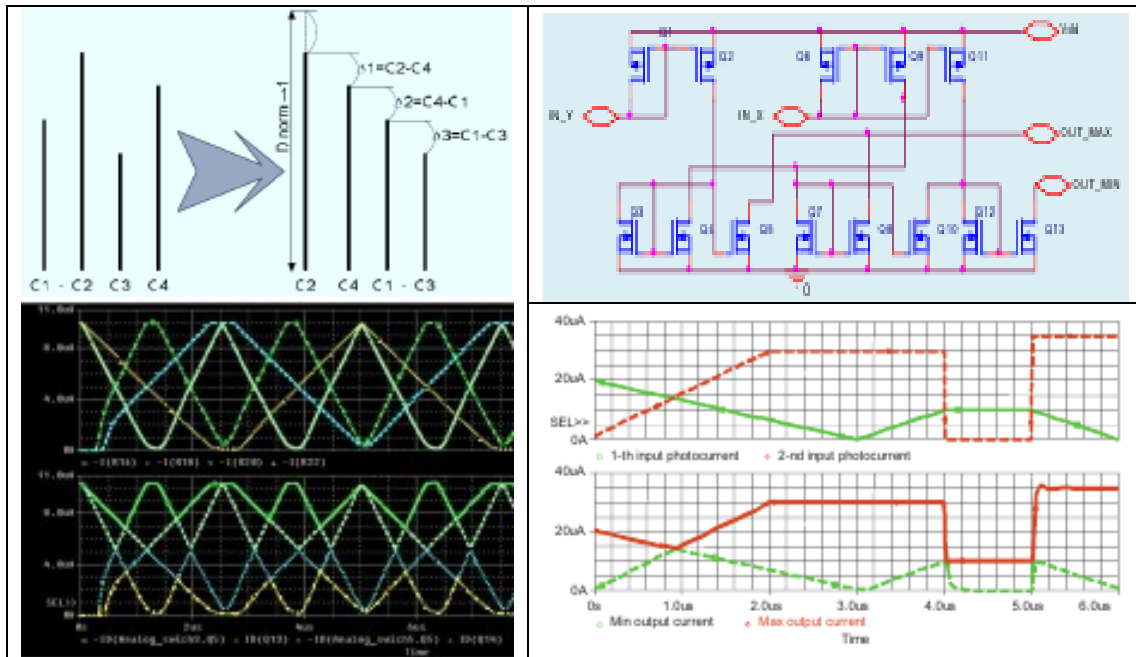


Fig. 3. Graphical representation of the SU operations (left top); Simulation results of the 4-input sorting unit of analog signals using of 6 the modified base analog cell based on two-input and two-output selector-rank disjunctive - conjunctive elements for small currents (bottom left), SRDCE circuit (top right) and its simulation results (bottom right)

The structural and internal circuit diagram of analog switchboard is shown in Fig. 2, 3. The block for difference and complement analog signals computation is not shown. Current mirrors circuits are used for design of such blocks. PSPICE simulation results of the processors with 4 inputs is shown in Fig. 5. The signal timing plots in Fig. 3, 5 confirm that the analog signal ranking circuits work correctly for both the 2-input and 4-input circuits. The essence of the proposed method is as follows. If we calculate the difference between the maximum and minimum outputs of the ranking scheme using the limited difference (LD) circuit based on CM, then it is equal to “nonequivalence”, and the complement to it is the “equivalence” from the input variables. Moreover, this complement is also calculated by the LD circuit based on CM. By combining (uniting) at the block output a certain subset of signals (currents) from the set of obtained differences between adjacent rank signals, we can easily form the required one or another output signal (current) corresponding to the selected CL function. Let's look at the simplest example. Let there are two input signals  $a_1$  and  $b_1$ . Then we can find:  $\max(a_1, b_1)$ ,  $\min(a_1, b_1)$ , and form the difference signals: 1)  $1 - \max(a_1, b_1)$ , 2)  $\max(a_1, b_1) - \min(a_1, b_1)$ , 3)  $\min(a_1, b_1)$ , the sum of 1)-th and 2)-th is equivalence ( $a_1, b_1$ ). The second difference is non-equivalence. In this case, these functions can be selected by the control vectors (1,0,1) and (0,1,0) of the multiplexer, and there are eight options in total. For the case with four inputs, for example, as in Fig. 2, 3, for inputs C1–C4 or  $(a_1, b_1, a_1^c, b_1^c)$ , after their ordering, there will be five difference signals at the multiplexer input. A 5-bit control vector will already create 32 new responses.

It is possible to realize processors for much more number of analog inputs by use of the considered approaches. It is important to realize such processors with 8 inputs, because of morphological image processing. The processors can operate with analog and time-pulse coded signals. Computation error does not exceed 1 %. The structure functions correctly, although error in the levels of signals before and after is 5–10 %. Input photocurrents lay in range 0.1 – 100  $\mu\text{A}$ , consumption power 1 – 10 mW. Thus analog structures not are very much effective for considerable  $n > 4$ . In our opinion, today it is difficult to say about the advantage due to the use of signals with time pulse coding, since the amplitude-analog approach may be more promising in terms of speed. It is necessary to look for other hybrid approaches as well.

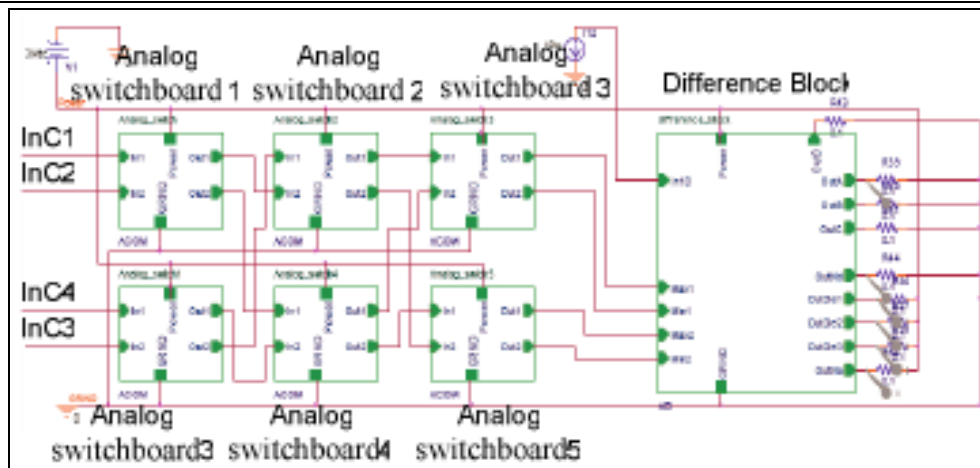


Fig. 4. The structural circuit of channel-nodes of element-wise analog processing, which was used for modeling in the Orcad Pspice environment

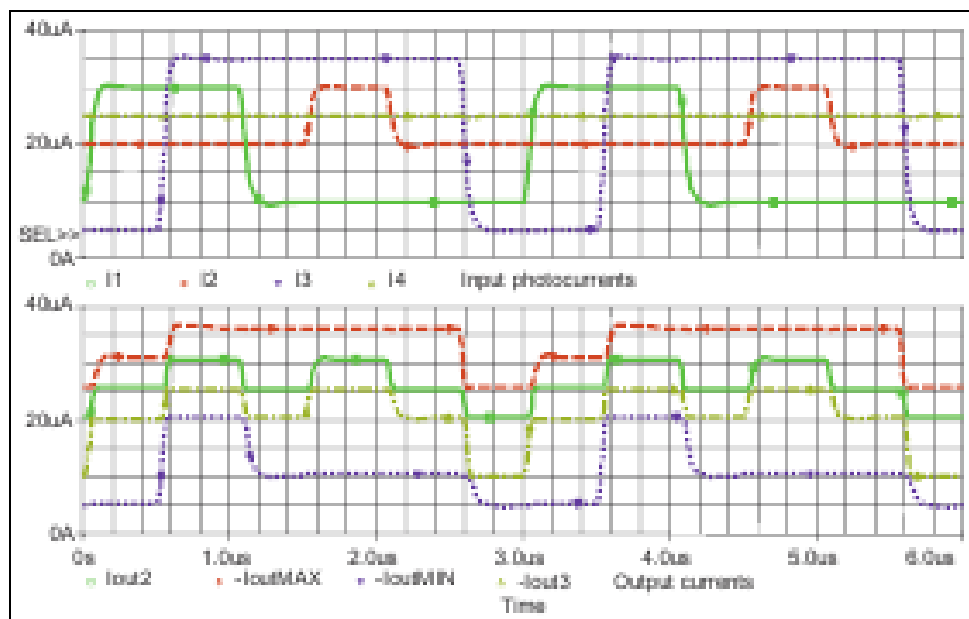


Fig. 5. Simulation results of the 4-input analog sorting structure

Examples of control vectors and a specific example of a window with signals to be processed, with the results obtained for it for different vectors with explanatory calculations, are shown in work [49]. It can be seen from them that a significant number of functions, operations from window signals, including any selected rank, the difference of the selected ranks, addition to the signal, weighted sums of the selected ranks, etc. can be generated at the processor output.

Thus, taking into account especially simple implementations of the operations of summation-subtraction, both for digital and analog signals, the proposed method significantly simplifies the implementation and extends the functionality, set of operations.

Taking into consideration the above-described approach, consisting in universality, let us recollect some known facts regarding the number of functions. The number of Boolean functions of  $n$  variables in algebra of two-valued logic (ATL), which is also Boolean algebra, equals  $2^{2^n}$ . In this ATL there are  $N_2 = 2^n$  atoms, which are minterms. Functions of  $n$  variables  $k$ -valued logic ( $k > 2$ ) are reflections  $A^n \rightarrow A$ , where  $A = \{0, 1, \dots, k-1\}$ , and the number of functions equals  $N_k = k^{k^n}$ . Algebra, formed by set  $^{\wedge}C_u = [0, 1]$  or  $^{\wedge}C_b = [-1, 1]$  is called CL algebra (ACL), and the number of CL functions, as reflections  $C_u^n \rightarrow C_u$  depending on the CL algebra can be infinite or finite (the set of reflections is always infinite). CL functions are called only those functions of the set  $N_{\wedge}$ , which are realized by formulas. The number  $N_{\wedge}$  of CL functions in the most developed CL algebra – quasi-Boolean Cleenee algebra ( $\Delta = (C_u, \wedge, \vee, -)$ ), in which any function on any set of arguments takes the value of one of the arguments or its negation, is finite. In this case the number  $N_{\wedge}(n)$  of functions of  $n$  arguments increases with increase of  $n$  very rapidly <sup>4</sup>:  $N_{\wedge}(0) = 2$ ;  $N_{\wedge}(1) = 6$ ;  $N_{\wedge}(2) = 84$ ;  $N_{\wedge}(3) = 43918$ . It should be noted that among fuzzy logics, considered in paper [6] and associated with three variations of fuzzy algebra, only one of them coincides with

Cleenee algebra, and the notion of fuzzy functions coincides with the notion of CL-function. That is why the sign of equality not always can be put between terms “continuous” and “fuzzy logic”.

Thus, our concept is to create elementary formal neurons that, through their tuning (training, but not training entire neural networks), are capable of performing any required mapping operators  $C_u^n \rightarrow C$ , even with a certain error.

In work [40] we proposed a new structure, which consists of optical node and a 2D array of equivalentors (**Eqs**) with optical inputs. Simulation on 1.5  $\mu\text{m}$  CMOS in different modes has shown that the equivalentors and their base units can operate correctly in low-power modes and high-speed modes, their energy efficiency is estimated to be not less than  $10^{12}$  an.op / sec per W the produced and can be increased by an order, especially considering FPAA [39]. Thus, at the inputs of each **Eq** we have two arrays of currents representing the compared fragment and the corresponding filter, and the output of the **Eq** is an analog signal, nonlinearly transformed in accordance with the activation function. And, as shown, the use of an array of cells that perform hardware, non-linear transformations adequate to auto-equivalence operations, even without WTA network allows the laborious computational process of searching for extremums in maps for clustering and learning not to be performed. Moreover, for the automatic selection of these extrema, only a few transformation steps are required. The results confirm the possibility of synthesizing cells with required accuracy characteristics of the transformation laws and, in particular, auto-equivalence functions, the microwatt level of power consumed by them, high speed. For the simplest and approximate approximation functions, but often quite sufficient for the selection of the winning function by the activation function, the cell circuits consist of only 17–20 transistors, have a very high speed ( $T = 0.25$  us), a small power consumption (less than 100 microwatts). The analysis of the results obtained confirms the correctness of the chosen concept and the possibility of creating a CDNE with generalized equivalence operations and vector processors of scalar relations, as hardware accelerators for compact high-performance machine vision systems, CNNs and self-learning biologically inspired devices.

### Conclusions

The paper proposes the mathematical foundations of design of CDNE with generalized equivalence neuro-fuzzy logic operations based on continuously logical cells (CLC) and current mirrors (CM), nodes of EM NN with functions of preliminary analogue processing for image intensity transformation for construction of analog and mixed image processors (IP) and neural networks (NN). The family of new operations “equivalence” and “non-equivalence” of neuro-fuzzy logic’s, which we have elaborated on the based of such generalized operations of fuzzy-logic’s as fuzzy negation, t-norm and s-norm are shown. Generalized rules of construction of new functions (operations) “equivalence” which uses operations of t-norm and s-norm to fuzzy negation are proposed. Despite the wide variety of types of operations on fuzzy sets and fuzzy relations and the related variety of new synthesized equivalence operations based on them, it is possible and necessary to select basic operations, taking into account their functional completeness in the corresponding algebras of continuous logic, as well as their most effective circuitry implementations. Among these elements the following should be underlined: 1) the element which fulfills the operation of limited difference; 2) the element which algebraic product (intensifier with controlled coefficient of transmission or multiplier of analog signals); 3) the element which fulfills a sample summarizing (uniting) of signals (including the one during normalizing). Several effective schemes of analog optoelectronic complementary-dual neuron-equivalentors as hardware accelerators SLECNS and their nodes have been developed and modeled. The proposed CDNE have a modular hierarchical construction principle and are easily scaled. Their main characteristics were measured. They have a processing-conversion time of 0.1–1  $\mu\text{s}$ , low supply voltages of 1.8–3.3 V, minor relative computational errors (1–5 %), small consumptions of no more than 1–10 mW, can operate in low-power modes less than 100  $\mu\text{W}$  and high-speed (1–2 MHz) modes. The relative to the energy efficiency of CDNE is estimated at a value of not less than  $10^{12}$  an.oper./sec. per W and can be increased by an order. The analysis of the obtained results confirms the correctness of the chosen concept and the possibility of creating CLCs for image intensity transformation and MIMO structures [13] on their basis, as hardware accelerators for compact high-performance systems of machine vision, CNN, SLECNS and self-learning biologically inspired devices with the number of such CDNEs equal to 1000, to realize the parallel calculation and nonlinear processing.

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