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USE OF SIMILARITY METRICS IN ROBUST TIME DELAY ESTIMATION

This paper addresses the task of time delay and direction of arrival estimation for a source of the wideband signal using two sensors with fixed displacement. The peculiarity of the task statement is that a limited time of signal observation is supposed and additive noise is assumed non-Gaussian with a heavy-tail distribution. This leads to a high probability of abnormal estimates for the conventional signal processing method based on cross-correlation. To decrease this probability, it is proposed to reformulate the task of cross-correlation processing to the task of similarity estimation between two data arrays. This allows using different similarity metrics, particularly those that have less sensitivity to outliers in data (impulse noise), and, thus, provide better robustness for non-Gaussian environments typical for several applications of time delay estimation. More than ten different similarity metrics are considered for the model of the symmetric α-stable distribution describing noise properties. It is shown that some metrics including cosine distance, Hellinger, and some others are able to provide sufficiently better accuracy of time delay estimation both in the sense of less RMSE of normal estimates and probability of abnormal estimates for typical values of α and a wide range of γ values for symmetric α-stable distribution.

Key words: time delay estimation, similarity metrics, robustness.

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ВИКОРИСТАННЯ МЕТРИК ПОДІБНОСТІ ПРИ СТІЙКОМУ ОЦІНЮВАННІ ЗАТРИМКИ ЧАСУ

У цій статті розглядається задача визначення часу затримки та напрямку приходу випромінювання від джерела широкосмугового сигналу за допомогою двох датчиків із фіксованою відстанню між ними. Особливістю постановки завдання є те, що вводяться припущення про обмеженого часу спостереження сигналу, а адитивний шум припускається негаусовим та таким, що має розподілом із важким хвостом. Це призводить до високої ймовірності аномальних оцінок для традиційного методу обробки сигналу на основі взаємної кореляції. Щоб зменшити цю ймовірність, пропонується переформулювати задачу взаємно-кореляційної обробки як задачу оцінки подібності між двома масивами даних. Це дозволяє використовувати різні метрики подібності, зокрема, ті, які мають меншу чутливість до викидів у даних (імпульсного шуму) і, таким чином, забезпечують кращу стійкість для негаусових середовищ, які є типовими для кількох застосувань оцінки часової затримки. Для моделі симетричного α-стабільного розподілу, що описує властивості шуму, розглянуто більше десяти різних метрик подібності. Показано, що деякі метрики, включаючи косинусну відстань, відстань Хеллінгера та деякі інші, здатні забезпечити суттєво кращу точність оцінки часової затримки як у сенсі меншого середнього квадратичного значення нормальних оцінок, так і ймовірності аномальних оцінок для типових значень α та широкого діапазону значень γ для симетричного α-стабільного розподілу.

Ключові слова: оцінка часової затримки, метрики подібності, стійкість.

Problem statement

The task of time delay estimation (TDE) or determination of the direction of arrival (DOA) based on TDE arises in many applications including teleconferencing [1, 2], seismology [3], hydroacoustics (passive sonars) [4], and others [5, 6]. For a simple (idealized) case of stationary wideband signal with the known spectrum, high signal-to-noise ratio (SNR), Gaussian additive noise with the known spectrum, fixed (not varying) time delay, and quite large observation time, the solution is known [1, 4]. It is based on cross-correlation processing of received signals with possible pre-whitening in time or spectral domain if additive noise is not white. The time delay is estimated by finding the largest (global) maximum of the cross-correlation function or its modification and then recalculated to DOA taking into account antenna geometry. However, the situation changes if one or several aforementioned assumptions are violated, i.e. if the signal source moves, SNR is small, and/or the noise is not Gaussian. Note that, for each particular application, there are different reasons to violate these assumptions. For example, additive noise is often non-Gaussian [7, 8], and/or a signal source is not stationary (its position changes in time) as this happens in teleconferencing and hydroacoustics. This leads to an increase in the variance of normal estimates of TDE and DOA as well as to a higher probability of abnormal estimates [9, 10]. This shows that it is desired to provide robustness of TDE in two senses [11], i.e. robustness with respect to the impulsivity of the noise and to limited a priori knowledge on statistics of the noise (parameters α and γ of the symmetric α-stable process simulating non-Gaussian noise).

Analysis of recent research

There are several ways to improve the performance of TDE and, in general, tracking a source of wideband signals. Shao and Nikias [12] proposed data processing based on fractional lower-order moments. Benesty et al [2] used minimum entropy to improve estimation. H. Li et al [13] concentrated on the estimation of the time delay derivative in a complex noise environment. Mehrjouyan and Alfi [14] applied an adaptive robust Kalman filter for tracking. A joint analysis of these publications shows that the robustness of estimation is tried to be improved at stages of primary signal processing [2, 12] and secondary processing of already obtained estimates [14, 15] or a set of sequentially obtained estimates of cross-correlation outputs [16]. In general, the performance improves due to better accuracy of elementary estimates obtained at the first stage (i.e. for short-time intervals of signal accumulation and processing) and more efficient (robust) post-processing of the already obtained time delay estimates or cross-correlator outputs. Then, it is reasonable to solve both subtasks. Recently, we have concentrated on improving the accuracy of TDE for elementary intervals looking for more efficient processing of signals embedded in non-Gaussian noise by partially or fully replacing the cross-correlation [17-19]. An approach based on robust DFT [20] was proposed in [17] but, unfortunately, the robust DFT has no fast algorithms and, thus, the computational efficiency of the corresponding processing is limited. Another approach [18] presumes the use of a center-weighted median filter [21] to remove impulses in received signals, but it is unclear what are the best parameters of such a filter depending on signal and noise properties. Finally, the authors of the paper [19] propose to apply similarity measures (metrics) instead of cross-correlation-based processing assuming that robust similarity measures or distances are able to provide desired properties to the entire processing. However, the amount of the similarity measures considered in [19] is very limited, and the final recommendations concerning the best among them are not given.

Thus, the Goal of this paper is the following: to investigate the possibility of using a wide set of similarity measures in time delay estimation for different possible practical conditions and to provide motivated recommendations on the best (most robust) similarity measures.

Analysis of possible solutions using different similarity measures

We assume that two sensors displaced by an a priori known distance *L* receive the mixtures of information wideband noise-like (WNL) signal and additive noise:

$$x_1(t) = s(t) + \xi_1(t), x_2(t) = s(t-\tau_0) + \xi_2(t) \quad x_1(t) = s(t) + \xi_1(t), \quad x_2(t) = s(t-\tau_0) + \xi_2(t) \tag{1}$$

where $s(t)$ $s(t)$, $t = [T_b; T_e]$ denotes the WNL information signal which is irradiated by a signal source); $\xi_1(t)$ and $\xi_2(t)$ denote the additive noise realizations for the corresponding sensors, τ_0 τ_0 is the true value of the time delay. There are the following assumptions on WNL signal: its mean is equal to zero and its spectrum is more "low-frequency" than the additive noise spectrum. For additive noise, we assume the following: the means (or location parameters) of the processes $\xi_1(t)$ and $\xi_2(t)$ are assumed equal to zeroes, they are characterized by probability density function that differs from Gaussian and it is more heavy-tailed although there is no absolutely accurate information about noise intensity and impulsivity. The latter assumption follows from practice where it is difficult to estimate statistical characteristics of the noise (especially, from its mixture with information signal) and, moreover, the noise statistics might vary in time. We denote the observation interval starting and ending time instances as T_b and T_e. It is also assumed that the maximal possible value of τ_0 determined by *L* and the wave propagation speed in a given medium *C* as $\tau_{max} = L/C$ $\tau_{max} = L/C$ is considerably smaller than $T_e - T_b$. Meanwhile, $T_e - T_b$ is also restricted from the upper side by the source motion, the computational efficiency of processing, and the necessity to get elementary estimates frequently enough. Because of this, $T_e - T_b$ is usually larger than τ_{max} by tens of times.

The conventional way of TDE presumes the calculation of the cross-correlation function:

$$Y(\tau) = \int_{-T/2}^{T/2} x_1(t) x_2(t+\tau) dt \tag{2}$$

where $T = T_e - T_b$ with the further search of the global maximum coordinate in the limits from $-\tau_{max}$ $-\tau_{max}$ to τ_{max} τ_{max}. This coordinate is accepted as the time delay estimate. Let us now rewrite the expression (2) as

$$E_1 + E_2 - 2Y(\tau) = \int_{-T/2}^{T/2} \left(x_1^2(t) - 2x_1x_2(t+\tau) + x_2^2(t+\tau) \right) dt \tag{3}$$

where $E_1 = \int_{-T/2}^{T/2} x_1^2(t)\, dt$ $E_1 = \int_{-T/2}^{T/2} {x_1}^2(t)\; dt$ and $E_1 = \int_{-T/2}^{T/2} x_2^2(t+\tau) dt$ $E_2 = \int_{-T/2}^{T/2} {x_2}^2(t+\ \tau)\; dt$ are, in fact, the energies of the mixture $x_1(t)$ $x_1(t)$ and $x_2(t)$ $x_2(t)$ [19]. Suppose that E_1 E_1 and E_2 E_2 are practically constant if WNL signal and additive noises are locally stationary. Then, it becomes clear that instead of searching for the global maximum of expression (2), one can search for the global minimum of (3) which is, in fact, the Euclidian distance between the received signals $x_1(t) x_1(t)$ and $x_2(t+\tau) x_2(t+\tau)$. For sampled versions of these signals $x_1(i), i=1,\dots,N\ x_1(i), i=1,\dots,N$ and $x_2(i+j), i=1,\dots,N\ x_2(i+j), i=1,\dots,N$, it is then possible to compute the similarity measure $S(j), i=-j_{\max},\dots,j_{\max}\ S(j), i=-j_{max},\dots,j_{max}$, where $j_{\max}\Delta t = \Delta\tau_{max} = L/C\ j_{max}\Delta t = \Delta\tau_{max} = L/C$ and Δt is the sampling rate.

The next step deals with the fact that Euclidian distance is not robust with respect to impulse noise. Then, some other similarity measures can be applied instead of it. In particular, it has been proposed in [19] to form the output as $S_\beta(j) = \sum |x_1(i) - x_2(i+j)|^\beta\ S_\beta(j) = \Sigma|x_1(i) - x_2(i+j)|^\beta$ where β denotes the order. Three values of β equal, namely, to 0.5 1.0, and 1.5 have been considered in [19] and it has been demonstrated that, depending on the additive noise impulsivity, all of these three values could be quasi-optimal in the sense of providing the best robustness. If so, other similarity measures that possess certain robust features can be potentially applied as well.

In our studies, we have used several similarity measures and distances [22]. One of them is Minkowski's distance expressed as

$$d_M = x_i - y_{i\,p} = \sqrt[p]{\sum_{i=1}^{n} |x_i - y_i|^p}\ , \tag{4}$$

where x and y are general notations for two vectors that can represent two (mutually shifted) signals received by two sensors for the task of our research, i is the vector element index, and p denotes the order. As is seen, in [19], we have used not directly the Minkowski distance but its analog. Also note that the Minkowski metrics with p=1 and p=2 are called Manhattan and Euclidian distances, respectively.

The Canberra metric [22] was originally used in solving cluster analysis problems, which involve assessing the similarity of data for grouping and building a hierarchy relative to the original data. This metric is written as

$$d_C = \sum_{i=1}^{n} |x_i - y_i| / (|x_i| + |y_i|) \tag{5}$$

A metric based on the Bray-Curtis (also called Sorensen) distance [22] has been mainly used for the assessment of the similarity of samples in botany. It is expressed as

$$d_{BC} = \sum_{i=1}^{n} |x_i - y_i| / \sum_{i=1}^{n} |x_i + y_i| \tag{6}$$

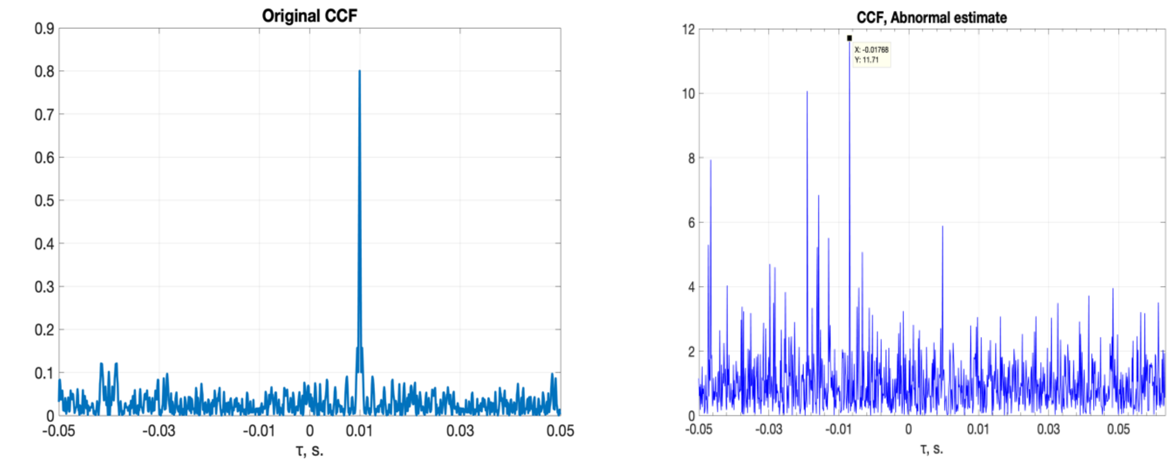
The Hellinger distance (see the expression below) as a modification of the Euclidian distance is intensively used in probability theory, cluster, and statistical analysis:

$$d_H = \frac{1}{\sqrt{2}} \sqrt{\sum_{i=1}^{n} \left|\sqrt{x_i} - \sqrt{y_i}\right|^2} \tag{7}$$

Instead of searching the maximum of the cross-correlation function like in the conventional method, we need to search for the global minimum of sampled functions (outputs) $S_{Mp}(j)$, $S_C(j)$, $S_{BC}(j)$, and $S_H(j)$ that are based on Minkowski metric with a given p, Canberra, Bray-Curtis, and Hellinger distances, respectively.

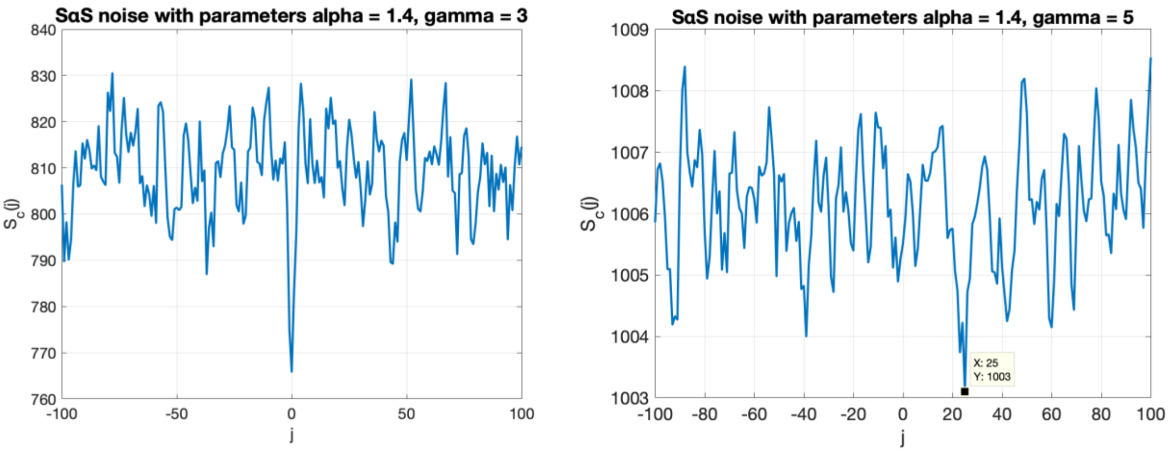
Let us give some examples to better explain the aforesaid. First, Figure 1 shows two examples of absolute values of the cross correlation function $Y(\tau)$ in case of using the standard approach. There is the situation of obtaining a normal estimate (τ_0 = 0.01 s) for a very high signal-to-noise ratio (Fig. 1a). In turn, the situation of abnormal estimation is presented in Fig. 1b. In this case, τ_0 is the same, but, due to heavy noise, the global maximum is observed for τ about – 0.007 s and the standard method fails.

Outputs for the considered distances differ from $Y(\tau_j) = Y(j), j=-j_{max}, \dots, j_{max}$ as well as between each other. To demonstrate this, Figure 2 presents the array $S_C(j), j=-j_{max}, \dots, j_{max}$, where j_{max} = 100 for the considered case. First of all, in this case, one needs to find the global minimum. The case of obtaining a normal estimate is depicted in Fig. 2,a where, in this case, τ_0 = 0 s and this corresponds to j = 0. As one can see, output values are of the order of hundreds and the minimal value for j = 0 is only slightly smaller than for other j. For more intensive non-Gaussian noise (see the details of the used noise model in the next section), the obtained estimate is abnormal – it is observed for j = 25, SNR is about -15 dB. The values of $S_C(j)$ due to more intensive noised (larger γ) have increased and they are all about 1006.



a b

Figure 1. Output CCFs *Y(τ)* using the standard approach for the situations of normal (a) and abnornal (b) estimation of time delay



a b

Figure 2. Output for the Canberra distance S_C(j): normal (a) and abnormal (b) estimation cases

Signal/Noise Model

The following similarity metrics are investigated in our study: Minkowski distance with p = 1.5, 1.0. and 0.5, Canberra, Bray-Curtis, and Hellinger distances. Simulations have been conducted to compare their effectiveness using a broadband noise-like signal. To model it, additive white Gaussian noise was passed through a low-pass filter to provide the upper (cut-off) frequency approximately three times lower than the Nyquist frequency (20 kHz in our case). The power (variance) was fixed and equal to one. To simulate additive noise independent for the receivers, a model of a symmetric alpha-stable (*SαS*) process [8] was chosen. This allows for the flexible variation of the noise intensity and distribution tail heaviness using two parameters: $\alpha_{S\alpha S}$ adjusts the distribution tail heaviness (smaller $\alpha_{S\alpha S}$ relates to heavier tails), and γ is the scale parameter responsible for the noise intensity (larger γ corresponds to more intensive noise). This makes it possible to vary the equivalent signal-to-noise ratio.

Statistical modelling was carried out using MATLAB in the following manner. 10,000 realizations of the noisy signal and noises $\xi_1(t)$ and $\xi_2(t)$ were generated for each considered pair of $\alpha_{S\alpha S}$ and γ in the first and second channels. Standard methods were used to obtain the output arrays for the standard approach denoted as Fourier approach in the plots below and new approaches based on the considered similarity metrics (distances). The root mean square error $\sigma_\tau(\gamma)$ of normal estimates and the probability of abnormal estimates $P_{abn}(\gamma)$ were determined for each of the considered methods. For both, the smaller the better. It should be noted that the latter criterion holds more significance.

Result Analysis

Simulation results were obtained for four $\alpha_{S\alpha S}$ values, namely 2, 1.8, 1.6, and 1.4, where $\alpha_{S\alpha S}$ = 2 corresponds to Gaussian noise.

The obtained plots $P_{abn}(\gamma)$ are presented in Fig. 3 for the proposed approach based on the Minkowski distance (recall that a larger γ corresponds to more intensive noise, to a smaller equivalent SNR, and to a larger probability of abnormal errors observed for all plots in Figures 3 and 4). For $\alpha_{S\alpha S}$ = 2 (Fig. 3a), the proposed approach performs slightly better than the conventional one (based on cross-correlation processing) for large values of γ. If $\alpha_{S\alpha S}$ = 1.8 (Fig. 3b), the advantages of using the proposed approach become obvious. The use of the

Minkowski distance with p=1.0 provides, on average, slightly better results than for other p values. For smaller $\alpha_{S\alpha S}$ values (Figures 3c and 3d), the conventional method fails for γ = 1.0. The proposed approach possesses certain robustness to the heavy-tailed noise even for γ = 2. The results for all values of p are approximately the same, so we can recommend using p = 1.

Let us analyze now the simulation data for three other distances. The results are presented in Fig. 4 for four values of $\alpha_{S\alpha S}$.

As one can see, the distances utilized in classification and cluster analysis are able to yield favourable outcomes. The standard approach based on cross-correlation processing yielded quite good results under ideal conditions ($\alpha_{S\alpha S}$ = 2, Fig. 4a). However, Hellinger and Bray-Curtis outperformed it. The use of processing based on the Canberra metric is not suitable for these conditions.

As noise impulsivity increases (see data for $\alpha_{S\alpha S}$ = 1.8, Fig. 4b), the conventional approach clearly becomes the worst ($P_{abn}(\gamma)$ values occur to be the largest for a wide range of γ variation). The data processing method based on Canberra distance is less efficient than for two other considered distances. The methods based on Hellinger and Bray-Curtis distances perform in a similar manner. For more impulsive additive noise (see data for $\alpha_{S\alpha S}$ = 1.6 and 1.4, Figures 4c and 4d, respectively), the conventional method fails even for γ = 1. Meanwhile, it is possible to state that the data processing method that relies on the Bray-Curtis metric performs in the best manner.

In practice, different situations are possible. First, one might know noise impulsivity (associated with $\alpha_{S\alpha S}$ for the *SαS* noise model) in advance. Then, the obtained results presented in Figures 3 and 4 allow for choosing the most appropriate similarity metric (distance). If $\alpha_{S\alpha S}$ is a priori known and it is difficult to measure it, a "robust" approach can be applied. In this sense, we recommend using either the Minkowski distance with the parameter p equal to unity or the Bray-Curtis distance. In the future, it seems possible to design some adaptive approach presuming estimation of non-Gaussian noise parameters and proper selection of distance.

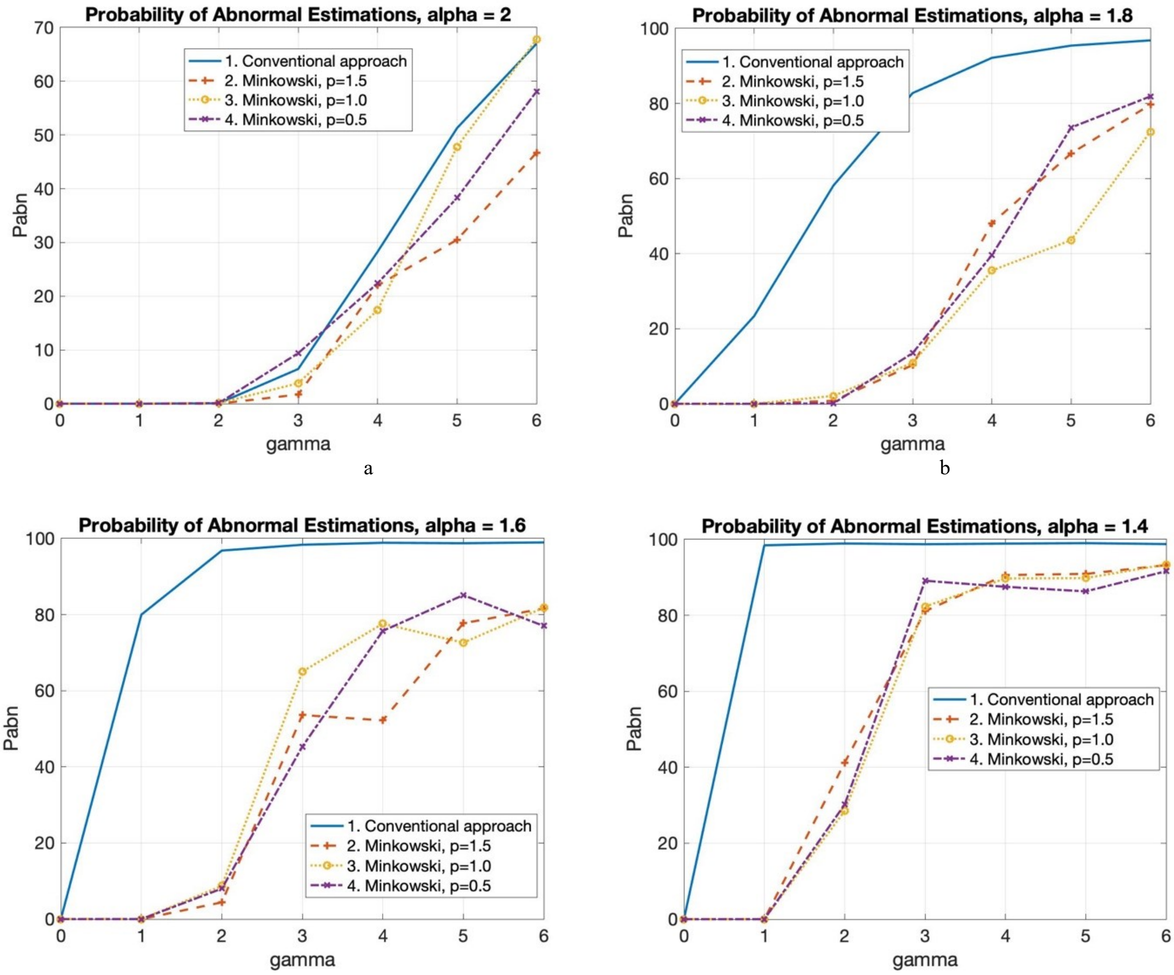


Figure 3. Comparison of performance for the conventional approach and the proposed approach based on Minkowski distance

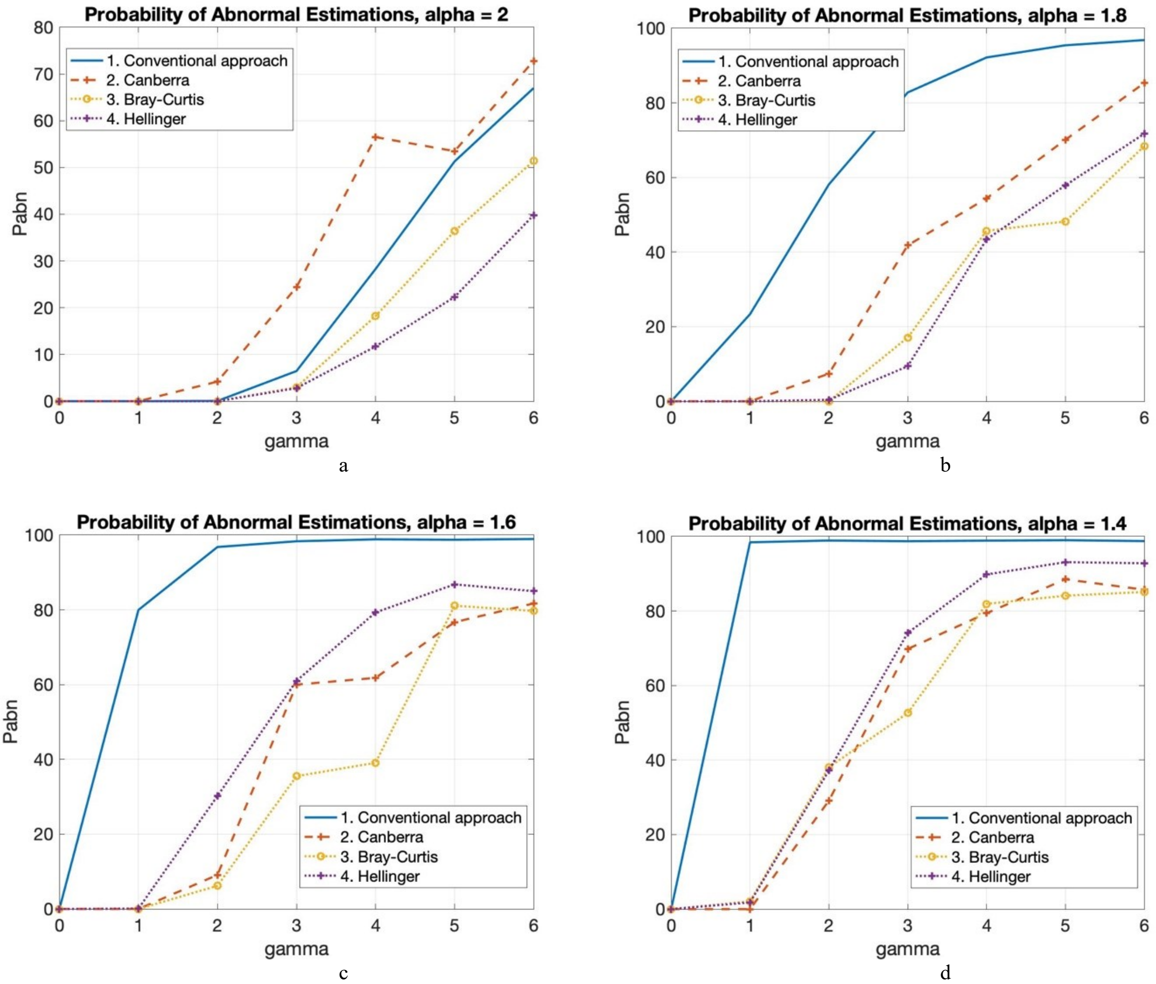


Figure 4. Comparison of performance for the conventional approach and the proposed approach based on Canberra, Bray-Curtis, and Hellinger distances

Conclusions

This article focuses on the application of similarity metrics in time delay estimation as an alternative to conventional cross-correlation processing, which serves as a reliable technique for TDE for WNL signals in favourable conditions of low-intensity Gaussian noise. It is shown that non-Gaussianty of the noise leads to a high probability of abnormal estimates of time delay. The proposed method relies on using robust similarity metrics as the basis of processing. Employing a robust distance metric as a CCF analogue provides rather fast and efficient processing for non-Gaussian environment. Simulation data giving some insight for choosing the most suitable metric based on noise parameters are presented. Through simulation results, it is evident that the proposed approach demonstrates a significant improvement compared to the conventional method. As a result, future research will aim to develop an adaptive metric that can be applied in practical situations.

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