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MODELING OF THE DYNAMICS OF ROTORS OF VARIABLE MASS OF TECHNOLOGICAL MACHINES

The dynamic parameters of rotors of variable mass of winding-unwinding machines in a non-linear setting were studied. For one of the approximations - the constant linear speed of the winding V=const, the engineering dependences on time for the following parameters were obtained in an explicit form: the current radius of the rotor r(t), the angular velocity ω(t), the moment of inertia I_x(t), the moment of the amount of movement L_x(t). These dependencies are the basis for creating a software-hardware complex of dynamic control and maintenance of specified parameters of unwinding-winding technological machines in online mode. Computer modelling of the model for the parameters of a specific technological machine - longitudinal cutting machine PRS5-201 manufactured by the Izhvazhmash plant was carried out. The simulation results were in good agreement with the experimental data obtained in the full-scale experiment. The relevance of the work is confirmed by acts of proceedings at several enterprises of the paper industry in Ukraine and Europe.

Keywords: rotor with variable mass, nonlinear dynamics, dynamic parameters, moment of inertia, moment of momentum.

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МОДЕЛЮВАННЯ ДИНАМІКИ РОТОРІВ ЗМІННОЇ МАСИ ТЕХНОЛОГІЧНИХ МАШИН

Досліджено динамічні параметри роторів змінної маси намотувально-розмотувальних машин в нелінійній постановці. Для одного з наближень - постійній лінійній швидкості намотування V=const отримано в явному виді інженерні залежності від часу для наступних параметрів: поточного радіусу ротора r(t), кутової швидкості ω(t), моменту інерції I_x(t), моменту кількості руху L_x(t). Дані залежності є основою для створення програмно-апаратного комплексу динамічного управління та підтримки заданих параметрів розмотувально-намотувальних технологічних машин в режимі on line. Проведено комп'ютерне моделювання моделі для параметрів конкретної технологічної машини – продольно-різального стану ПРС5-201 виробництва заводу «Іжважмаш». Результати моделювання отримали гарну збіжність з експериментальними даними, отриманими в натурному експерименті. Актуальність роботи підтверджена актами провадження на кількох підприємствах паперової галузі в Україні та Європі.

Ключові слова: ротор зі змінною масою, нелінійна динаміка, динамічні параметри, момент інерції, момент кількості руху.

Introduction

Modern processing technologies use high-performance winding and unwinding machines for compact packaging and processing of tapes, strips, sheets, cables, ropes, fabrics and non-woven materials, etc. High productivity requires the maximum possible speeds of the winding-unwinding process, which, in turn, is limited by the mechanical characteristics of the wound materials. On the other hand, the presence of dynamic models of these processes helps to increase the productivity of technological processes, and the control of the transition to rigid modes with limited acceleration allows not to exceed the dynamic loads and the ultimate limit of strength of the wound material, which also improves the quality of technological operations.

The aim of the work is the construction of dynamic models of rotors with variable mass at a constant linear speed of winding-unwinding, as a theoretical basis for the creation of a software and hardware complex for controlling the parameters of the technological process, the identification of the regularities of the dynamics of rotors of variable mass, the construction of a mathematical model of a two-port rotor of variable mass in a non-linear setting and experimental verification of proposed models.

Review of recent publications.

Rotary machines are extremely common in many fields of mechanical engineering, mining and processing, agricultural machinery, metalworking, food and light industry. At the same time, in many cases, the technological processes of processed masses of raw materials interacting with the rotors of technological machines lead to a change in the mass and geometry of the rotors Meshchersky I.V. [1], Kosmodemyanskyi A.A. [3], Popikov O.O. [4].

Among the works devoted to the dynamics of bodies of variable mass, the classical works of I.V. Meshcherskyi, I.I. Artobolevskyi, M.E. Zhukovskyi, I.I. Blekhman, A.A. Kosmodemyanskyi, L.G. Loitsyanskyi, A. I. Lurie, F. R. Gantmacher, R. Appell, E. Routh, J. Pollard, and others. It is known Meshchersky I.V. [1], Kosmodemyanskyi A.A. [2], Popikov O.O. [4], that rotors with variable mass have variable inertial parameters and variable geometry. Studies Bykov V.G.[5, 6, 8], Kovachev A.S. [9], Zaitsev N.N. [11, 10] are devoted to the regularities of the dynamics of rotors of variable mass in a nonlinear setting. A significant part of research is devoted to the problems of balancing, changing resonance frequencies, minimizing the dynamic reactions of supports, and identifying the limits of the asymptotic stability of rotor dynamics Kurakin A.D. [15], Degtyarev S.A. [16], Horbenko A.N. [17, 18, 19]. Models of the dynamics of rotors in a nonlinear setting with infinitely large turns are given in Sorokin F.D. [13, 12]. In Kurakin A.D. [15], Degtyarev S.A. [16], Makarov A.A.[14], the peculiarities of the nonlinear dynamics of rotary machines with a controlled change of friction in the supports, impulse contact interaction with the stator and the presence of damage such as cracks are indicated. The influence of the anisotropy of moments of inertia and the definition of critical modes of nonlinear dynamics are given in Horbenko A.N. [19], Volkova N.V. [20], Gorovoi, S.A. [21]. Possibilities of dynamic monitoring of rotors of compressors of Ukraine's state-run transmission system are studied in Krivoruchko V.T. [22], and accelerated resource forecasting of such systems is given in Grabar I.G. [24,25]. Reduction of vibrations in rotors using passive balancing devices and their mathematical modeling, as well as a four-ball balancer design proposed by Makram M. [23]

Despite the large number of publications devoted to the dynamics of rotors with a variable mass, the issues of modeling the dynamics of rotors with a variable mass in an explicit form, with obtaining convenient engineering formulas, remain poorly studied. Such models make it possible to propose a strategy for managing the operating modes of rotary machines and significantly increase their productivity and product quality. Therefore, the study of the dynamics of technological machines with rotors of variable mass is interesting and relevant from both a theoretical and a practical point of view.

Formulation of the problem.

Variable mass rotors are quite widespread in modern technological and transport machines (rolls, drums, spindles, coils for winding or unwinding threads, ropes, tapes), heavy engineering, power engineering, instrumentation, etc.(1-4). A fundamental feature of such problems is the variability of mass and geometry parameters over time. Dynamics, vibrations, change of natural frequencies, minimization of negative consequences, minimization of dynamic components in supports, etc. In modeling the dynamics of rotors with a change in mass, it is conditionally possible to distinguish three classes of problem formulation:

1. *V=const* – constant linear speed of winding (unwinding) tape (strip)
2. *ω=const* – constant angular speed of the rotor of the winding-unwinding machine
3. *V(t)=var*; *ω (t)=var* is the general case.

Theoretical studies. In this work, we will use the constraint 1 - *V=const*. As shown above, the rotors of winding and unwinding machines, tapes, threads, ropes, rolls change their mass *m*, radius *r*, moment of inertia I_x over time:

$$\begin{gathered} m = var \\ r = var \\ I_x = var \end{gathered} \qquad (1)$$

As a result, variable torque M_{kp} is required for the drive shaft, such rotors, with variable angular velocity *ω* and variable value of power *N*:

$$\begin{gathered} M_{кр} = var \\ \omega = var \\ N = var \end{gathered} \qquad (2)$$

The differential equations of motion of the rotor can be written in a general form (in a linear formulation):

$$\ddot{\varphi}_x I_x = M_{кр} - M_m \qquad (3)$$

where M_T is the reduced moment of friction.

To solve (3) under general initial conditions:

$$\begin{gathered} \varphi_x /_{t=0} = \varphi_0 \\ \omega_x /_{t=0} = \dot{\varphi}_x /_{t=0} = \dot{\varphi}_{x0} \end{gathered} \qquad (4)$$

it is necessary to define functional dependencies:

$$m(t); r(t); I_x(t) \qquad (5)$$

It follows from (5) that with variable *Ix(t)* the differential equation of the rotor motion (3) is only a partial case, that is, an approximation for $I_x \approx const$.

For the general case of the problem formulation, we will use one of the central theorems of dynamics - the theorem on the change of the momentum of the amount of motion:

$$\frac{d\vec{L}}{dt} = \vec{M}^{e} \tag{6}$$

For rotors with two fixed wheelbarrows, that is, a fixed axis of rotation *OX*, the projection of the vector equation (6) onto this axis gives:

$$\frac{dL_x}{dt} = M^{x} \tag{7}$$

Where:

$$L_x = I_x \cdot \omega_x ; M^{x} = M_{кр} - M_{m} \tag{8}$$

It follows from (7)-(8) that it is possible to build a mathematical model of the dynamics of the variable mass rotor, if it is possible to build the functional dependencies (5).

Case 1. *V=const*

Let the speed of the winding tape:

$$V = \frac{dl}{dt} = const \tag{9}$$

where l(t) is the length of the tape.

Case (9) corresponds to the conditions of minimizing the effects of jerks and shocks in the system, which means minimizing changes in the tension force *T(t)* of the winding (unwinding) tape or web. In the case of collecting straw into a roll (9) corresponds to the condition of the movement of the baler at a constant speed, which from a practical point of view is also quite convenient for practical implementation.

Assumptions of the model: Let a tape of thickness *h* be fed to a drum of radius r_0 at a constant speed *V*.

Determine $r(t)$; $\omega(t)$; $\varphi(t)$; $I_x(t)$ in an explicit form as a function of time.

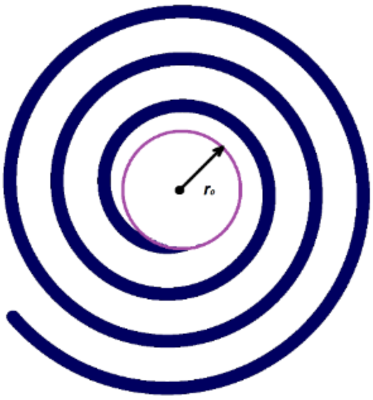

Fig. 1. To the derivation of the dependencies $M_{пр}(t)$ and $\omega(t)$

As follows from Fig. 1, the current radius *r* and the length of the *i*-th turn l_i can be expressed:

i	r_i	l_i	(10)
1	$r_1=r_0+h$	$l_1=2\pi(r_0+h)$	
2	$r_2=r_0+2h$	$l_2=2\pi(r_0+2h)$	
…	…	…	
n	$r_n=r_0+nh$	$l_n=2\pi(r_0+nh)$	

Then the total length of the wound tape

$$l_{\Sigma} = \sum_{i=1}^{n} l_i = 2\pi\left[\underbrace{\left(r_0 + r_0 + r_0 + \ldots + r_0\right)}_{n\ разів} + h\left(1+2+3+\ldots+n\right)\right] = \\ = 2\pi\left[nr_0 + h\frac{n+1}{2}\cdot n\right] = 2\pi n\left[r_0 + \frac{h}{2}(n+1)\right] \tag{11}$$

This allows you to determine the number of turns *n*, as

$$n = \frac{r_{max} - r_0}{h} ; nh = r_{max} - r_0 \tag{12}$$

It follows from (11)-(12).

$$l_\Sigma = \frac{2\pi}{h}\left(r_{max} - r_0\right)\left(r_0 + \frac{r_{max} - r_0 + h}{2}\right) = \\ = \frac{2\pi}{h}\left(r_{max} - r_0\right)\left(\frac{r_{max} + r_0 + h}{2}\right) \tag{13}$$

For $h<<(r_{max}-r_0)$ it follows from (13).

$$l_\Sigma \approx \frac{2\pi}{h}\left(r_{max} - r_0\right)\left(\frac{r_{max} + r_0}{2}\right) \approx \frac{\pi}{h}\left({r_{max}}^2 - {r_0}^2\right) \tag{14}$$

(14) allows you to find the total time of the winding roll t_Σ:

$$t_\Sigma = \frac{l_\Sigma}{V} = \frac{\pi}{hV}\left({r_{max}}^2 - {r_0}^2\right) \tag{15}$$

As in the case of (10), when V=const, it allows to calculate ωi and Δt_i for each turn

i	ω_i	Δt_i	
1	$\omega_1 = \frac{V}{r_0 + h}$	$\Delta t_1 = \frac{2\pi}{\omega_1} = \frac{2\pi(r_0 + h)}{V}$	(16)
2	$\omega_2 = \frac{V}{r_0 + 2h}$	$\Delta t_2 = \frac{2\pi}{\omega_2} = \frac{2\pi(r_0 + 2h)}{V}$	
3	$\omega_3 = \frac{V}{r_0 + 3h}$	$\Delta t_3 = \frac{2\pi}{\omega_3} = \frac{2\pi(r_0 + 3h)}{V}$	
...	...	...	
N	$\omega_n = \frac{V}{r_0 + nh}$	$\Delta t_n = \frac{2\pi}{\omega_n} = \frac{2\pi(r_0 + nh)}{V}$	

The values of ω_n and Δt_n make it possible to obtain the functional dependence of $\omega(t)$ without the parameter n. However, before that, it is necessary to obtain the sum of $\Delta t_{i\Sigma}$ in order to obtain the total time from the start of winding to reaching the i-th turn:

Therefore :

i i	Δt_i	$t_{i\Sigma}$	
11	$\Delta t_1 = \frac{2\pi}{V}(r_0 + h)$	$t_{1\Sigma} = \frac{2\pi}{V}(r_0 + h)$	(17)
22	$\Delta t_2 = \frac{2\pi}{V}(r_0 + 2h)$	$t_{2\Sigma} = \Delta t_1 + \Delta t_2 = \frac{2\pi}{V}\left(r_0 + h\right) + \frac{2\pi}{V}(r_0 + 2h)$	
33	$\Delta t_3 = \frac{2\pi}{V}(r_0 + 3h)$	$t_{3\Sigma} = \Delta t_1 + \Delta t_2 + \Delta t_3 = \frac{2\pi}{V}\left[r_0 + h + r_0 + 2h + r_0 + 3h\right]$	
….	...	...	
TN	$\Delta t_n = \frac{2\pi}{V}(r_0 + nh)$	$t_{n\Sigma} = \Delta t_1 + \Delta t_2 + \ldots + \Delta t_n = \frac{2\pi}{V}\left[nr_0 + \frac{n + nh}{2}\cdot n\right] = \frac{2\pi}{V}n\left[r_0 + \frac{h}{2}(n+1)\right]$	

The current time (17) and the value of ωn from (16) allow us to write the dependence between ω_n and $t_{\Sigma n}$ in a parametric form:

$$\begin{cases} \omega_n = \dfrac{V}{r_0 + nh} \\ t_{\Sigma n} = \dfrac{2\pi}{V}n\left[r_0 + \dfrac{h}{2}(n+1)\right] \end{cases} \tag{18}$$

From the second equation of system (18), it is easy to obtain the equation with respect to n:

$$n^2\frac{h}{2} + n\left(r_0 + \frac{h}{2}\right) - t\cdot\frac{V}{2\pi} = 0 \tag{19}$$

The solution of which gives

$$nh=\sqrt{\left(r_0+\frac{h}{2}\right)^2+\frac{hV}{\pi}\cdot t}-r_0-\frac{h}{2} \tag{20}$$

Substitute nh into the first equation of system (18):

$$\omega(t)=\frac{V}{\sqrt{\left(r_0+\frac{h}{2}\right)^2+\frac{hV}{\pi}\cdot t}-\frac{h}{2}} \tag{21}$$

(21) allows you to functionally set the dependence of the angular speed of the rotor on the time *t*, which starts from the beginning of winding the roll. For the thickness of the wound layer $h << r_{min}$, the value *h/2* is neglected, as it is infinitely small of a higher order of smallness. Then:

$$\omega(t)=\frac{V}{\sqrt{r_{min}{}^2+\frac{hV}{\pi}\cdot t}} \tag{22}$$

Table 1 shows the estimate of the approximation error (22)

h/r_{min}	0,1	0,01	0,001	0,0001	0,00001
δ, %	5%	0,5%	0,05%	0,005%	0,0005%

It follows from (21)-(22) that the angular velocity of the rotor is inversely proportional to the winding time to the power of 0.5. Approximation (22) allows you to obtain engineering formulas for calculating other rotor parameters from time. In particular, the dependence on time of the current radius of the rotor:

$$r\left(t\right)=\sqrt{r_{min}^2+\frac{hV}{\pi}t} \tag{23}$$

variable mass of the rotor from the time of winding:

$$m\left(t\right)=m_0+V\rho_1\cdot t \tag{24}$$

moment of inertia of the rotor from the time of winding:

$$I_x\left(t\right)=\frac{\left[m_0+V\rho_{\Pi}\cdot t\right]\left[r_{min}^2+\frac{hV}{\pi}t\right]}{2} \tag{25}$$

moment of the amount of movement of the rotor from the time of winding:

$$L\left(t\right)=I_x w=\frac{V}{2}\left[m_0+V\rho_{\Pi}t\right]\sqrt{r_{min}^2+\frac{hV}{\pi}t}\ - \tag{26}$$

derivative *dL/dt*:

$$\frac{dL}{dt}=\frac{V}{4}\cdot\frac{2V\rho_{\Pi}\left(r_{min}^2+\frac{hV}{\pi}t\right)+(m_0+V\rho_{\Pi}\cdot t)\frac{hV}{\pi}}{\sqrt{r_{min}^2+\frac{hV}{\pi}t}} \tag{27}$$

In this way, the approximation (22) - (27) was obtained - the functional dependence of the dynamic parameters of the rotor on the winding time. These dependencies are completely determined by the winding speed *V=const*, the tape thickness h and the drum radius r_{min}. The obtained dependencies (22) - (27) allow solving engineering problems of the dynamics of a rotor of variable mass when winding (unwinding) a layer of constant thickness with a constant linear speed of winding.

As a result of the research, a mathematical model of the dynamics of the rotor of variable mass was built and the regularities of its movement in a wide range of parameters were revealed - when the mass of the rotor and its moment of inertia change by 10-100 times. This makes it possible to carry out a dynamic analysis of the movement of the rotor and to choose rational technological parameters of the machine, as well as to propose a methodology for choosing the optimal modes of operation of units with rotors of variable mass, in particular - for unwinding and winding machines according to the criterion of the limit stresses in the winding material (rope, tape, canvas) . On the basis of these models, algorithms and instrumental tools for the design of rotary machines operating under conditions of changes in the mass-dimensional and inertial parameters of the system are proposed. The proposed models allow working out optimal control modes, in particular, in a significant part (more than 80%) in a computer experiment, significantly automating engineering calculations, creating a CAD subsystem "Dynamic analysis and control of rotary machines of variable mass and dimensional characteristics" and the corresponding hardware and software complex. This is relevant both at the design stage and at the stage of operation of such machines. The obtained results make it possible to improve the calculations of the dynamics of rotors of variable mass and to determine the limit modes of operation of the units according to the specified criteria.

Application example.

The rotor of the longitudinal cutting state has the following parameters:

$r_{min}=0{,}15\ m;\ \ r_{max}=0{,}9\ m;\ \ \ B=1{,}8\ m;\ \ \ h=10^{-4}\ m$

$\rho_1=0{,}144\ kg/m^1\ ;\ \rho_2=0{,}080\ kg/m^2\ ;\ \rho_3=800\ kg/m^3\ \ ;\ \ \ V=5\ m/s$

$l=(\ r^2_{max}\ -\ r^2_{min})*\pi/h=24739\ m$

$m_{var}=B*L*\rho_2\ \ =\ \ 3562\ kg;\ \ \ m_0 = \pi*\ r^2_{min}\ *B*\ \rho_{steel}\ \ = 1018\ kg$

$m_{\Sigma rotor}\ = m_0 + m_{var}\ \ = 4580\ kg;\ h/r_{min} =10\text{-}4/0{,}15=0{,}00066.$ then $\delta =0{,}033\ \%.$

The simulation data of the dynamic parameters of the rotor are shown in Fig. 2-Fig. 7.

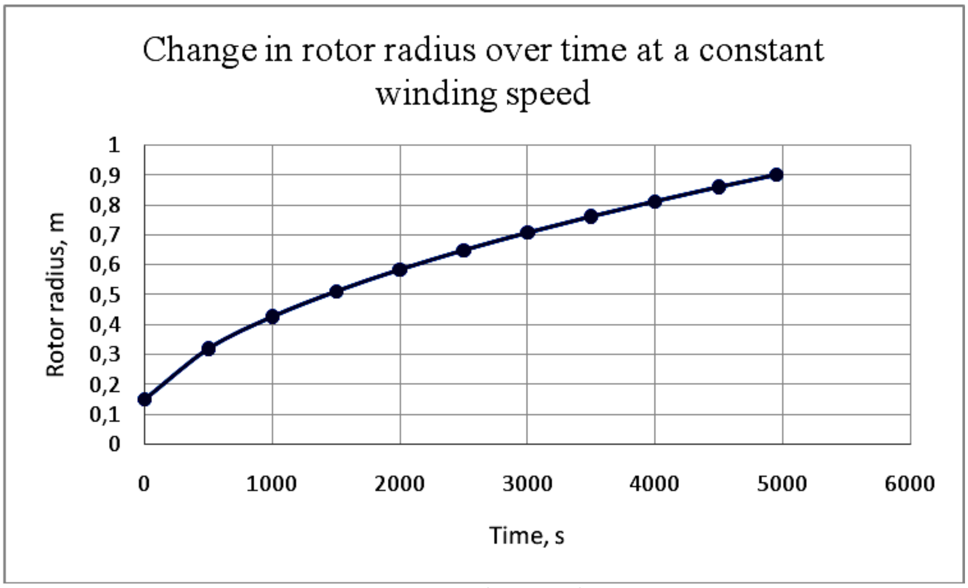

Fig. 2. Change in rotor radius as a function of winding time

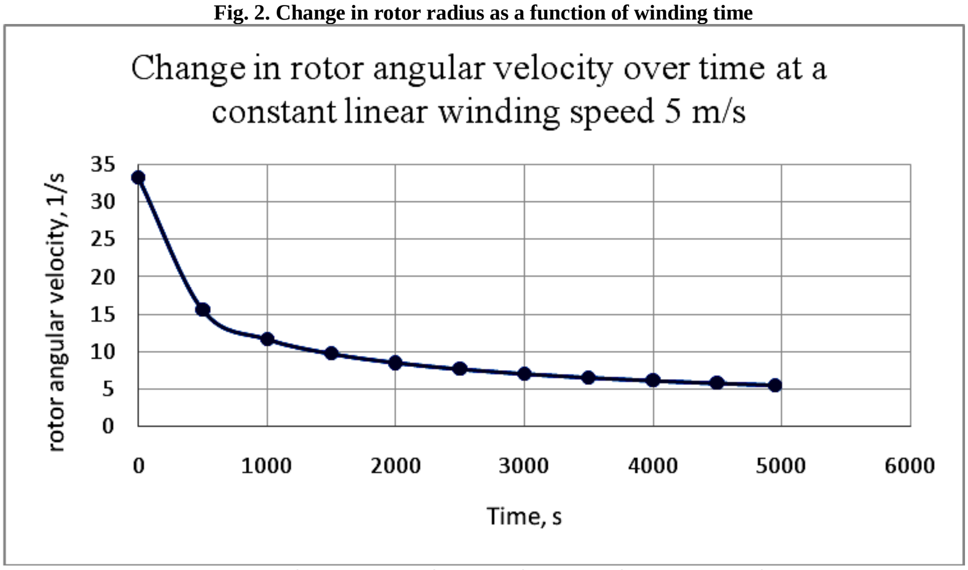

Fig. 4. Dynamics of the moment of inertia of the rotor from the time of winding

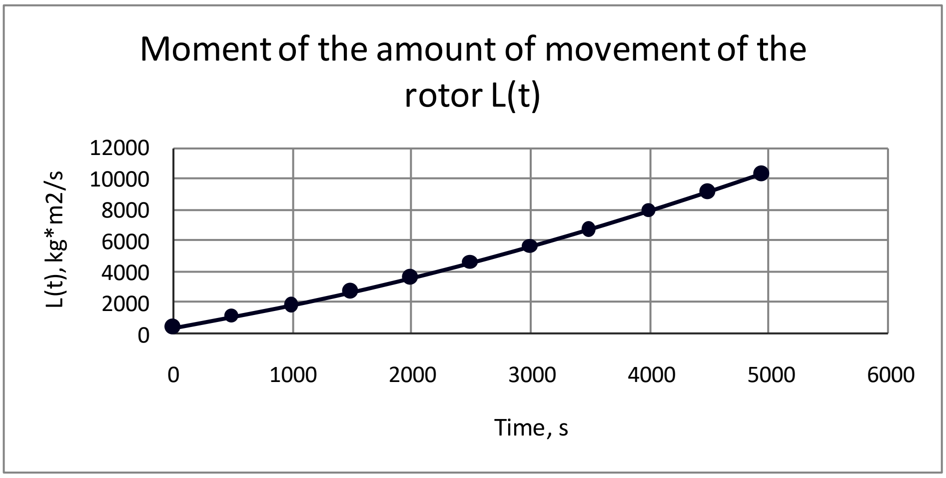

Fig. 5. Dynamics of the moment of the amount of movement of the rotor from the time of winding

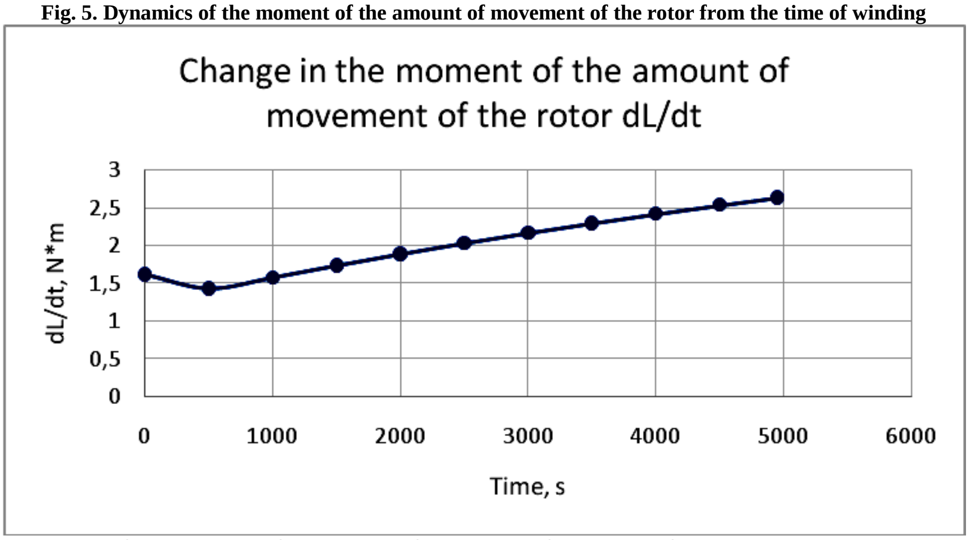

Fig. 6. Dynamics of the derivative of the moment of the amount of movement of the rotor during the winding process

Conclusions

1. As a result of the simulation, the equations of the dynamics of the variable-mass rotor as a function of the winding (unwinding) time were obtained in an explicit form in a nonlinear formulation.

2. Obtained engineering formulas for changing the winding radius, angular velocity of the rotor, moment of movement and its derivative as explicit functions of time, which allow to program the monitoring and control system for the process in real time (on line).

3. On the example of specific technological equipment - a longitudinal cutting machine - the proposed models were tested and implemented in production at several paper enterprises in Ukraine and Europe

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